TRANSACTIONS OF
THE
OPTICAL SOCIETY

VOL. XXVII. 1925-26 No. 5.

FRAUNHOFER'S WORK AND ITS PRESENT-DAY SIGNIFICANCE

BY PROFESSOR MORITZ VON ROHR, PH.D., HON. M.D.

MS. received, 17th July, 1926. Read (in abstract) and discussed, 24th June, 1926.

ABSTRACT. The hundredth anniversary of the death of Fraunhofer has presented a fitting opportunity for giving a more accurate account of the advances he made in optical work than has hitherto been possible. This account is based on his own papers, contemporaneous descriptions, and recent detailed measurements of his telescopes. Since various old English achromatic telescopes were also available for measurement, it has been possible to compare the results of the London and Benediktbeurn-Munich products. Whilst the English opticians—in particular J. Ramsden and the firm of Dollond—are renowned owing to their successfully carrying out experimental methods which enabled them reasonably to correct for spherical aberration, Fraunhofer obtained markedly superior results by the application of scientific methods of measurement, testing, computation, and glass-making. A description of the development of his work in Vienna is given and attention is drawn to his three main achievements as indicated by M. Faraday and more fully outlined by E. Abbe.

POSITION OF THE OPTICAL INDUSTRY AT THE BEGINNING OF THE 19TH CENTURY

England

IN order to appreciate Fraunhofer's work we must try to picture the position of the optical industry as far back as the beginning of the 19th century. English optical goods were certainly in great demand; of the compound instruments we must especially mention the telescopes with draw-tubes. In that war-stricken period they were much sought after and it is interesting to recall the fact that the form of the terrestrial telescope "with brass drawers" had been introduced for military purposes by the old and reputed firm of Dollond. Messrs Zeiss have been fortunate in obtaining for their historical collection a Dollond telescope with vellum draw-tubes and covered with nurse skin (Fig. 1). As I have attempted to prove

elsewhere (1), I think that it was made about 1780. Recently Dr Kessler has measured the objective. It is a doublet, as Fig. 2 shows, the full particulars, reduced to a focal length of 500 mm., being given under the figure. The glasses have the values $n_D = 1{\cdot}52776$, $\nu = 58{\cdot}4$ and $N_D = 1{\cdot}58419$, $\nu = 39{\cdot}7$, which correspond fairly closely to those of present-day types. Fig. 3 shows, in the generally accepted

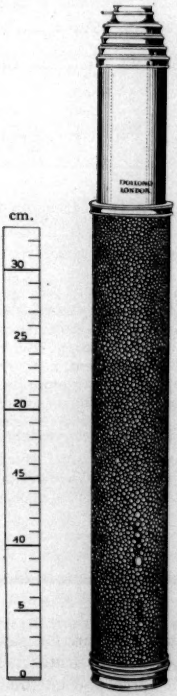

Fig. 1. Dollond telescope.

Fig. 2. Objective of Dollond telescope. Actual size for $f_1' = 500$ mm. Radii, thicknesses, and separation in mm.

$r_1 = +117{\cdot}7$ $\quad d_1 = 2{\cdot}19$
$r_2 = -140{\cdot}9$ $\quad b = 0{\cdot}04$
$r_3 = -122{\cdot}2$ $\quad d_2 = 2{\cdot}16$
$r_4 = +397{\cdot}0$

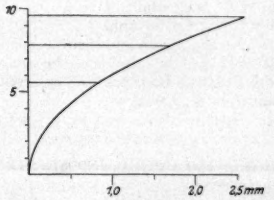

Fig. 3. Spherical aberration of Dollond objective.

manner, the spherical aberration of the direct pencil. It will be seen that there is distinct over-correction. Later on the English opticians were able with this simple type of objective to reduce, but not to eliminate, the over-correction, as Fig. 5, which is reproduced from Ref. (2), shows.

Towards the close of the 18th century the optician Jesse Ramsden (b. 1735, d. 1800) apparently enjoyed a remarkable reputation, and we know that Thomas Young had a very exalted idea of Ramsden's ability. Fortunately the historian W. Kitchiner (b. 1777, d. 1827) gave his very best attention to the development of the telescope and later on we shall cite his last work, published a short time before his death, as it gives us a very important list of the more outstanding telescope makers then working in London.

About 1800, as an anonymous author (3) tells us in a book of the year 1811 (it was probably written by the same W. Kitchiner), the London makers were rather depressed by the difficulty of obtaining useful pieces of crown and flint glass of reasonably large diameter. It appears that a certain glass factory, the "Ratcliffe works," ceased to be of any assistance and in consequence nobody was able to make telescopes approaching a diameter of 5 inches (12·5 cm.), which was the

Fig. 4. Objective of Berge telescope. Actual size for $f_1' = 500$ mm. Radii, thicknesses, and separation in mm.

$r_1 = +128{\cdot}357$

$d_1 = 2{\cdot}63$

$r_2 = -247{\cdot}682$

$b = 0{\cdot}16$

$r_3 = -195{\cdot}712$

$d_2 = 1{\cdot}90$

$r_4 = +474{\cdot}699$

$s_D' = 494{\cdot}014$

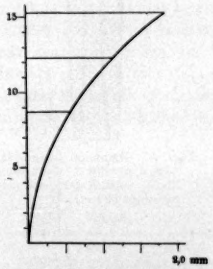

Fig. 5. Spherical aberration of Berge objective.

largest size the firm of Dollond had been able to produce, and that only on one occasion. That the available glass was generally scarce we may further conclude from the fact that, as our anonymous author tells us, London opticians were loath to bring out telescope objectives as doublets instead of triplets like the first ones made by J. Dollond from 1758 onwards. The spherical correction of the doublets was reputed to be much inferior to that of the triplets.

As no data with regard to the instruments of that period were known to me, I was very pleased when, in 1918, I had the opportunity of buying for the Zeiss historical collection an old military telescope made by Berge, Ramsden's successor, as he styled himself on the instrument (2). The general plan and the spherical aberration are shown in Figs. 4 and 5 in the same way as these particulars for the Dollond telescope are given in Figs. 2 and 3. The glasses have the values $n_D = 1{\cdot}5325$, $\nu = 57{\cdot}8$ and $N_D = 1{\cdot}5991$, $\nu = 38{\cdot}0$, which correspond fairly closely to the previous ones. From the data given I think we may safely conclude that

the instrument with a doublet as objective was sold in London about 1805. I was also able to buy two smaller Ramsden telescopes in Königsberg a little later and the objective of one of them turned out to be very well preserved. It was a triplet (Fig. 6) by this splendid maker and it gave me the long-sought opportunity of comparing the two forms after careful measurements of radii, thicknesses, separations, and refractive indices. I am indebted to my friend and colleague, Dr H. Kessler, for all these data. Fig. 7 shows that, in comparison with the simpler type, the Ramsden triplet objective is decidedly better corrected; doubtless the Dollond triplet objectives are also better corrected, but measurements of these have not yet been made. For the crown glass we have assumed the same type as is used in

Fig. 6. Ramsden objective. Actual size for $f_1' = 500$ mm. Radii, thicknesses, and separations in mm.

$r_1 = +283{\cdot}136$

$d_1 = 3{\cdot}93$

$r_2 = -379{\cdot}104$

$b_1 = 0{\cdot}30$

$r_3 = -288{\cdot}947$

$d_2 = 3{\cdot}35$

$r_4 = +252{\cdot}587$

$b_2 = 0{\cdot}25$

$r_5 = +306{\cdot}531$

$d_3 = 4{\cdot}22$

$r_6 = -379{\cdot}551$

$s_D' = 495{\cdot}109$

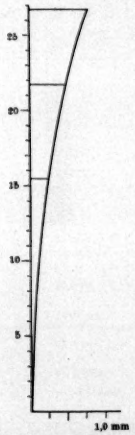

Fig. 7. Spherical aberration of Ramsden objective.

the Berge telescope, and measurement of the flint glass gave values very near to $N_D = 1{\cdot}5961, \nu = 38{\cdot}3$. The trigonometrical computation of these carefully measured objectives afforded such a remarkable corroboration of the old description that I think it worth while to give the results according to my system, which has been widely accepted for photographic objectives. I must mention that I decided (3), in conformity with the suggestion of my late lamented colleague, Dr Erfle, to take 500 mm. (about 20 inches) as unit focal length. Thus the diameters in the reduced formula can conveniently be shown without any reduction in size. It is at once obvious that the construction is very simple (collective lenses bi-convex, dispersive lenses bi-concave), that the triplet has by far the better correction although its relative aperture is larger than that of the doublet, but that the spherical aberration

of the direct pencil is not fully removed, the spherical aberration of the eyepiece being still smaller.

I had the Berge eyepiece (Fig. 8)—a four-lens combination as was usual at that time—measured also and was able to give in Ref. (2) quite a number of results of the calculations. I shall, however, restrict myself here to the astigmatic aberrations, the principal rays crossing at the anterior vertex of the objective as the

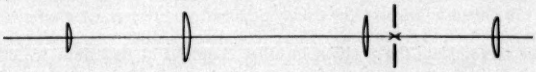

Fig. 8. Berge eyepiece. Half actual size for $f_3' = 25$ mm. Radii, thicknesses, and separations in mm.

$r_1 = \infty$ $r_2 = +203{\cdot}3$ $r_5 = +36{\cdot}8$ $r_7 = +24{\cdot}67$

$d_1 = 1{\cdot}7$ $d_2 = 3{\cdot}1$ $d_3 = 1{\cdot}5$ $d_4 = 2{\cdot}1$

$r_3 = -18{\cdot}08$ $r_4 = -30{\cdot}78$ $r_6 = -95{\cdot}0$ $r_8 = -184{\cdot}0$

$b_1 = 50{\cdot}2$ $b_2 = 77{\cdot}7$ $b_3 = 13{\cdot}0$ $b_4 = 43{\cdot}2$

$n = 1{\cdot}51$ $s_1 = -21{\cdot}42$ $s_8' = \infty$

middle of the entrance pupil of the whole instrument (Fig. 9). It may be remarked that from measurements of focal length the glass appears to correspond to O. Schott's Ocularkron O 60 as regards the colour correction in Fig. 10. For my diagram I reduced the eyepiece to 25 mm. (about 1 inch) so that the telescope had a magnification of exactly 20 times. Unfortunately the length of the terrestrial eyepiece is so considerable that I was obliged to draw it to one-half of its actual

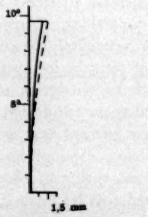

Fig. 9. Astigmatic aberration of Berge eyepiece. The full curve shows the results for the sagittal and the broken curve for the tangential bundles of rays.

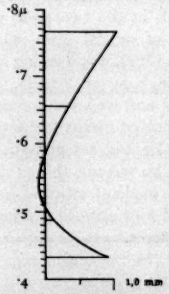

Fig. 10. Secondary spectrum of Berge telescope.

size (Fig. 8). From Fig. 9 it is obvious that the state of the oblique pencils is quite satisfactory and the reputation of these telescopes as good instruments is not unmerited.

Fig. 10 shows the secondary spectrum of the whole telescope (in the front focal plane of the eyepiece). The chromatic correction of the whole instrument is very remarkable in both cases and we may safely assert that even to-day we could not obtain a better chromatic correction with the same glass at our disposal.

Summing up we may say that Ramsden's methods of trial and error—no others were possible at that time without exact measurements of refraction and dispersion—allowed him to reduce the spherical aberration of the direct pencil to a fair amount and to correct the chromatic errors in a very satisfactory way. I may mention at the same time that he could educate his journeymen most effectively, as I hope to show before this Society in a paper to be read later.

France

If we contemplate the position in other countries at that time, we certainly find optical artists of great perseverance and industry in Paris, but they were still more hampered by the want of optical glass than their London brethren. Evidently the demand of all the opticians in these cities was so inconsiderable that no glass factory was induced to go in specially for optical glass, as the expensive experiments would not have been remunerative even if the owner had been exceptionally successful.

In both the western countries this difficulty of improving optical glass seems to have been admitted, and state boards and commissions were not stinting in offering considerable rewards to enterprising glass specialists. Incidentally the French offers are known to me in much greater numbers than the English. Possibly some one versed in English technical history may know of more English glass rewards than the single one of 1789 that is known to me(4).

Although in the two great western countries no progress was made in the production of optical glass, a very energetic outsider, Pierre Louis Guinand (b. 1748, d. 1824), was working in the duchy of Neuchâtel, then under the government of the King of Prussia. Quite recently reliable information(5) has been forthcoming and we know for certain, and with knowledge of many details, that this man without means had worked with great energy and high inventive faculty at the task he had set himself. His pecuniary success was as yet only inconsiderable as he was not able to do away with striae, although he had invented the stirring rod made of crucible clay as early as the spring of 1805. It is probable that he sold flint glass to spectacle factories and that he himself made spectacles and perspectives.

Germany

No optical industry worth mentioning was found in Germany: if we remember J. H. Tiedemann of Stuttgart we name probably the only German optician of some small reputation for achromatic telescopes. As I have not so far been able to buy one of his instruments, I cannot give reliable information as to the perfection of his workmanship.

When we reflect on this unsatisfactory condition of affairs we cannot wonder at some enterprising German journeyman trying to work with London optical masters and by this means getting a better education in his handicraft. We know for certain that young Johann Friedrich Voigtländer of Vienna(6) adopted such

a course and that in 1808 he established himself in Vienna as an optician after some years of work with London masters. Incidentally it has not been possible as yet to find out the names of any of his masters. As I hope to show next winter, he made good use of the time he spent on his optical education and it is quite interesting to watch the development of his works and to find out how he introduced English methods to Viennese workmen. It was not until immediately after 1808 that he was able to develop his business because he then found a much stronger competitor in a Bavarian factory, the history of which will interest us during this article.

During the year 1802 the want of modern surveying instruments (7) was much felt in Bavaria and during the next few years three men, Reichenbach, Utzschneider, and Liebherr, formed a company for making philosophical instruments of a higher class. Whereas Georg Reichenbach (b. 1772, d. 1826) and Joseph Liebherr (b. 1767, d. 1840) were well-trained mechanics, Joseph Utzschneider (b. 1760, d. 1840) not only provided the necessary funds but, as a successful and enterprising business man, undertook the optical branch, although he evidently had no particular knowledge in this respect.

By means of exploratory travels through all the modest optical factories in South Germany Utzschneider soon found that the greatest difficulty was in the provision of optical glass, especially flint glass. With the help of Swiss friends he was put into communication with P. L. Guinand, the only authority on optical glass proper at that time. After some hesitation on both sides he engaged him in September 1805, especially as Guinand seemed to him to be an expert at grinding lenses and finishing perspectives and telescopes as well.

Utzschneider's idea was at first not only to erect a factory for optical glass at his property at Benediktbeurn, an old monastery, but also to found optical works (independently of the company) in order to make spectacles, perspectives, and telescopes (8).

The progress during the years 1805–7 seems, however, to have been rather slow as the glass factory did not easily yield the necessary raw material. It also seems that Guinand was too much occupied with glass-making so that his superintendence of the grinding and polishing of lenses was nearly impossible. It appears that in 1807 the company had a small optical establishment in Munich under J. Niggl as foreman, but the extension was quite inconsiderable compared with the great mechanical institute under Reichenbach's special supervision.

Fraunhofer's Career

In the same year (1807), however, J. Fraunhofer was engaged as a journeyman optician and he had naturally at first to work under Niggl's direction. Joseph Fraunhofer (b. 1787, d. 1826) was the eleventh and last child of a master glazier in the townlet Straubing. He soon lost both his parents and after a very incomplete education was put out in 1798 as an apprentice to the Munich master mirror-maker P. Weichselberger. He was to serve six years as an apprentice and all that we know of his master does not give us a favourable impression of his ability. During

his apprenticeship he received some help from educated men; an astronomer, U. Schiegg, a former friar, seems to have furthered him in a small way. It seems that he very soon drew Reichenbach's attention to himself and in the end of the year 1807 he was transferred with the whole optical concern to Benediktbeurn, where probably Guinand had the supervision of this department. It is possible that Niggl soon quarrelled with Guinand; he certainly left soon after and Fraunhofer was optical foreman when he had not quite finished his 21st year. Unfortunately we do not know any details of his first year as foreman; we must only conclude that he was very successful, as in February 1809 he was one of the leading directors and his name appears in the firm. It is needless to say that such an advancement of a journeyman in his 22nd year cannot be accounted for but by very great achievements. It is very probable that during these 14 months or so he was particularly successful in devising grinding and polishing machines as well as instruments for examining and controlling the already ground and polished surfaces in order to approach as nearly as possible to the ideal spherical form. By far the most important achievement was his method, based on the idea of Newton's rings, of applying test surfaces. As to his authorship of it there is no doubt whatever, but as to the time of its introduction we can only surmise that it was between 1807 and February 1809. I may point out that he did not strive to emulate the English, then the best, standard, but that he aimed at reaching the ideal form as closely as scientific means would permit.

Glass-making

Judging from later explanations we must conclude that even Guinand's glass was far from homogeneous and Fraunhofer seems to have found early that very great accuracy was necessary if trigonometrical calculation was to be used for planning beforehand the whole objective. Anybody more closely interested in the development of his views might consult two of my papers (7, 8), in which the traces left in his writings have been put together.

In any case he soon found it necessary to learn the art of glass-making, and from August 1809 he was instructed in Guinand's methods by the inventor himself. As here again the different steps in the development are now lost to us, everything of this nature being considered by Guinand as well as by Utzschneider and Fraunhofer as strictly secret, we are compelled to draw conclusions, although in this way we can only obtain probable results.

When, however, we learn that Utzschneider, the proprietor of the glass factory in Benediktbeurn, gave the order in September 1811 that Fraunhofer was henceforth to direct the glass-making, we must conclude that he had mastered the difficulties during the preceding two years in a way that the proprietor considered satisfactory. From September 1811 up to December 1813 we find Fraunhofer as well as Guinand busy melting optical glass. We must therefore consider the progress so made as the scientific and technical property of both these inventors and decline to ascribe it to one of them separately. That this was Abbe's standpoint in such questions admits of no doubt, but readers may be still more interested in

the fact that in 1862 G. G. Stokes held a similar opinion*. When we consider, therefore, that not only the Benediktbeurn works were able from 1811–13 to melt glass discs of greater and greater diameter, but also that P. L. Guinand himself was in a position, from 1815 onwards, to make his glass factory at les Brenets more and more a paying establishment (5, p. 195), which it was not up to 1805, we must look to the years 1811–13 as being the commencement of this successful development.

Refractive Index Measurements

There can be no doubt that Fraunhofer tried with all his power to find out a successful method of measuring the refractive indices for distinct radiations of the spectrum. Whoever is interested in this progress will find the data collected as completely as possible in Ref. (8). We may safely conclude that he was able in the autumn of 1813 to find exact refractive indices for six different wave-lengths and that probably in the next spring he succeeded in using the dark lines of the solar spectrum† for this purpose.

By these means he was not only able to measure the mean refraction, say of the *D* lines, and ascertain with very great exactness the index of the mean refraction at any chosen place of the crucible, but he studied as well the dispersion through the whole visible spectrum. The characteristic changes of dispersion had been conjectured by such early scientists as Clairaut and Boscovich, and R. Blair of Edinburgh had in 1792 brought the importance of such studies to the knowledge of practical opticians, but all this work was rather vague and uncertain as the refractive indices could then not be measured with anything like certainty.

Dispersion

As soon as Fraunhofer had removed this difficulty, he attacked the problem of dispersion. As he was interested in it as a manufacturing optician he chose the following form. Given a certain part of the spectrum, say between *C* and *D*, and designating the indices of the crown and flint by n and N respectively, he formed the expression $(D, C) = \frac{N_D - N_C}{n_D - n_C}$. As he used for dividing points the dark lines

* "...Any one who has worked in concert with another zealously engaged in the same research must have felt the benefit arising from the mutual interchange of ideas between two different minds. Suggestions struck out by one call up new trains of thought and fructify in the mind of another; whereas they might have remained barren and unfruitful in the mind of the original suggester. The benefit of cooperation is by no means confined to the carrying out, according to a preconcerted plan, of a research involving labour rather than investigation; it is felt in a most delightful form in the prosecution of original investigations." 32nd Meeting of Brit. Assoc. Cambridge (1862). Notices and Abstracts, p. 1.

† Seven of these lines had been already observed by W. H. Wollaston, but this eminent philosopher failed to make use of his discovery for optical measurements. It is therefore quite logical that, as far as these lines are employed for measuring purposes, they are called Fraunhofer's lines. Fraunhofer, by the way, found within the easily visible part of the spectrum not less than 574 of these lines. For particulars of Wollaston's discovery see Ref. (7), p. 541.

B, C, D, E, F, G, H, he could form six expressions for (. , .) and show that the values for

	(*C*, *B*)	(*D*, *C*)	(*E*, *D*)	(*F*, *E*)	(*G*, *F*)	(*H*, *G*)
Flint 13 and water were	2·562	2·871	3·073	3·193	3·460	3·726
and for Flint 13 and oil of turpentine were	1·868	1·874	1·783	1·843	1·861	1·899

He concluded from these values, which were found by exact measurement*, that it might be possible to select the components of the crucible in such a way as to bring about a notably greater proportionality, and he succeeded so far as the pair Flint 13 and Crown *M* 1·667 1·704 1·715 1·737 1·770 1·816 shows. Crown *M* was, however, not durable.

For ordinary glasses with disproportionate dispersion a doubt arose as to which of these relations should be chosen in order to effect the best achromatism. In his endeavour to answer this question by scientific means he had to find out the numerical values of the brightness over the whole visible solar spectrum. He solved in a masterly manner the then quite new problem of heterochromatic measurement and could decide the above question by pure calculation. As he was not quite satisfied with the results so obtained, he deliberately varied the value for the pair of glasses chosen, taking care, however, that the different objectives so constructed were equally free from spherical aberration and only differed as regards the chromatic correction. The ultimately selected value was somewhat smaller (1·98) than the amount (2·012) at which he arrived theoretically.

In any case we may agree that he did his utmost to lay down beforehand the details of the whole objective so that the grinding, polishing, and mounting journeyman had nothing to do but to work to the data prescribed by the calculating optician. For that time it was quite a new method of procedure and in fact the only one possible in order to compete successfully with English opticians. In such a way he fought the personal art and ability of artists like J. Ramsden by means of the arms of applied science.

It is quite natural that the telescopes of the Munich Institute were much appreciated in Germany as well as in Austria. For astronomers in general such a high valuation was brought about by the great refractor finished for Dorpat in the year 1824. Struve's enthusiastic report with regard to the superior design and the masterful execution of this instrument was of decidedly great value for the Munich works. The free diameter was 9 inches (24·5 cm.) and it seems as if the parallactic mounting was then generally considered a novelty.

The echo of Struve's report is of specially great importance for our present object. We learn from David Brewster's remarks, when he vividly complained that the superiority of this branch of mechanical art had been taken away from

* He gives six decimal places, but according to Dr H. Kessler the results can safely be taken only to five decimal places.

England, that Fraunhofer's methods—they seem to have been overlooked in England during the first peaceful years after the Napoleonic wars—made a very great impression on technical experts.

Fraunhofer's Instruments

I shall now refer to Fraunhofer's instruments, of which we have measured two objectives (both doublets) and one eyepiece. In the case of the objectives we must distinguish two types. Figs. 11 and 12 give the results for an objective from a

Fig. 11. Fraunhofer objective (1). Actual size for $f_1' = 500$ mm. Radii, thicknesses, and separation in mm.

$r_1 = +213{\cdot}930$ $\quad d_1 = 3{\cdot}91$

$r_2 = -218{\cdot}675$ $\quad b = 0{\cdot}378$

$r_3 = -215{\cdot}555$ $\quad d_2 = 3{\cdot}85$

$r_4 = -10062{\cdot}3$

Fig. 12. Spherical aberration of Fraunhofer objective (1).

Fig. 13. Fraunhofer objective (2). Actual size for $f_1' = 500$ mm. Radii, thicknesses, and separation in mm.

$r_1 = +365{\cdot}8$ $\quad d_1 = 3{\cdot}35$

$r_2 = -146{\cdot}0$ $\quad b = 0{\cdot}022$

$r_3 = -148{\cdot}8$ $\quad d_2 = 2{\cdot}29$

$r_4 = -526{\cdot}3$

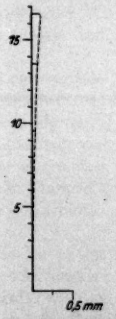

Fig. 14. Spherical aberration of Fraunhofer objective (2).

Fig. 15. Fraunhofer eyepiece. Half actual size for $f_s' = 25$ mm. Radii, thicknesses, and separations in mm.

$r_1 = \infty$ $r_2 = \infty$ $r_3 = +35{\cdot}36$ $r_4 = +22{\cdot}68$
$d_1 = 3{\cdot}3$ $d_2 = 2{\cdot}32$ $d_3 = 5{\cdot}64$ $d_4 = 3{\cdot}15$
$r_5 = -28{\cdot}55$ $r_6 = -36{\cdot}69$ $r_7 = \infty$ $r_8 = \infty$
$b_1 = 74{\cdot}6$ $b_2 = 137{\cdot}83$ $b_3 = 61{\cdot}38$
$s_8' = \infty$

hand telescope, for which $f' = 54$ cm. The arrangement of the lenses—they were separated by a brass ring—shows that the telescope was not of the very first quality. The crown lens was nearly equi-convex and the flint lens nearly plano-concave. The spherical aberration is, however, so small that I have shown in the broken curve the error $f_0' - \frac{h}{\sin u'}$. From this it is obvious how far Abbe's sine condition, to use the present-day notation, has been fulfilled. The glasses have the values $n_D = 1{\cdot}52658$, $\nu = 60{\cdot}2$ and $N_D = 1{\cdot}63383$, $\nu = 34{\cdot}6$.

In the other objective (Figs. 13 and 14) the two lenses are separated by small pieces of tin foil of equal thickness. The glasses are very similar to the previous ones and have the values $n_D = 1{\cdot}52862$, $\nu = 59{\cdot}5$ and $N_D = 1{\cdot}63532$, $\nu = 34{\cdot}5$. The lens forms are decidedly more curved than the previous ones, the result of this as regards the correction being such that the spherical aberration for D cannot be distinguished on the usual scale from the ideal case of a perpendicular straight line. The fulfilment of the sine condition (in the present-day notation) is very satisfactory; I may remark that probably Fraunhofer paid attention, by trigonometrical computation, to the images of non-axial points, but did not know our expression for the sine condition.

The Fraunhofer eyepiece (Fig. 15) for the objective of Fig. 11 is very long in comparison with that of Berge; in consequence it is shown to half scale. The glass has the value $n_D = 1{\cdot}529$. The correction for oblique pencils is very satisfactory, although a greater angle ($u' = 18°$) was taken for calculation (Fig. 16). I do not know how Fraunhofer was led to this form, whether by calculation or trial. I would only mention that according to Thomas Young's work of 1800–1, the formulae for exact computation had already been developed. However, I do not think that Fraunhofer knew Young's work.

Fig. 16. Astigmatic aberration of Fraunhofer eyepiece. See Fig. 9.

In concluding these remarks on the results of the calculations I would like to express my indebtedness to my colleagues H. Boegehold, A. König, and W. Merté for the computation and diagrams.

W. Kitchiner's book (9) gives us many details of London-made telescopes for these very years. We see from the price lists (pp. 24–5 and 84–5) that G. Dollond then offered achromatic telescopes with diameters from 3·8 inches (9·6 cm.) downwards, whereas C. Tulley mentioned in his list the greater diameter of 5 inches (12·7 cm.) as well; his largest achromatic telescope (with Guinand's flint disc) of 7 inches (17·8 cm.)* was evidently a unique achievement and looked upon as such.

When we compare with this the Munich price list of 1st November, 1820, we find there ready-made objectives for mechanicians from 6 Parisian inches (16·2 cm.) downwards, whereas achromatic refractors with free diameters of 6·5 and 8·5 Parisian inches (17·6

* More exactly 7¼ inches (18·5 cm.) according to *Phil. Mag.* 67 (1826) 377–82, 791–2.

and 23 cm.) were offered for sale from 1st September, 1816. To-day we must agree with Brewster's opinion that English opticians at that time had overlooked the progress made in Munich.

Fraunhofer's Character and Achievements

One of the greatest authorities of that period on the working of large optical surfaces, John F. W. Herschel, had called on Fraunhofer in Munich and in August, 1825, expressed his appreciation of his modesty as well as of his talents (10, p. 111). This appreciation is the more convincing as he was obliged by filial piety to oppose in the first part of this letter some of Fraunhofer's statements as to the efficiency of large reflectors. I do not know in which year before 1825 Herschel had been to Munich, but it seems possible that the refusal of the Glass Committee of the Royal Astronomical Society to decide the priority of invention of optical glass was influenced or brought about by Herschel, then an important member of this Committee. He may have heard from Fraunhofer himself that such a decision of a technical question would be the more difficult as two inventors working together at the same task were concerned in bringing the art of glass melting to the standard of 1813 and later.

In any case it was J. F. W. Herschel, I should think, who informed M. Faraday with regard to Fraunhofer in such a way that he was able in 1829 to give his short but masterful appreciation of Fraunhofer's achievements in the optical line.

On the other hand we must refrain from taking for granted that this appreciation of Fraunhofer's work was by any means universal. J. South, an astronomical observer of very high repute at that time, was, for instance, so satisfied with the astronomical instruments he found in Paris—in the shops of the older Lerebours and of Cauchoix—that he did not think it advisable to study personally Fraunhofer's instruments by making a journey to Munich.

We may therefore assume that the short duration of Fraunhofer's life was against his appreciation in London: he had not time enough to fight successfully against such disadvantages as the great distance and the difference of languages certainly caused. When he died, his successors were only able to repeat his designs for new instruments and they do not seem to have made great exertions to make themselves known in the English market.

Curiously enough they never tried to sell flint and crown glass in a raw state (8, p. 288), but strictly upheld Utzschneider's plan of 1805 to melt glass for their own use only. This decision was the more injurious to the Benediktbeurn glass works as these willingly restricted themselves in this way to a small size, whereas the demand of foreign glass buyers would have brought them much more work and the incentive of competition. Their position at the time of Fraunhofer's death was very promising, as Aimé Guinand was not very energetic in his glass factory, also the Company formed by Guinand's widow and Daguet was—up to 1834–5 when the principal part of the concern was transferred to Solothurn—of a very modest extension, and the Parisian factories belonging to Henry Guinand and to

G. Bontemps do not seem to have prospered. It is only about the year 1839 that the Parisian glass experts attained the same eminence as the works at Solothurn, which were much more closely connected with the decisive technical development in Benediktbeurn between 1809 and 1813.

Fraunhofer's Influence on Later Opticians

When Fraunhofer died, his work was much studied and in particular the leaders of the optical branch of the Vienna Polytechnic Institute, J. J. Prechtl (11) and S. Stampfer (12, 13), tried their best to teach the Viennese opticians—in the first place J. Voigtländer and G. S. Plössl—the art of making good telescopes. Stampfer's two papers of 1828 are still well worth reading, and it may be interesting for opticians to hear that Stampfer was then evidently aware of the aberration that Abbe later on described as chromatic difference of spherical aberration.

It might be quite useful to compare the way in which Airy (14) and Coddington (15) tried to help opticians with Prechtl's and Stampfer's method and this might be the more interesting as all their papers were written nearly at the same time and were undoubtedly independent of one another.

Later on, in 1856, when C. A. Steinheil successfully tried to bring back to his Munich Institute the scientific renown belonging to Fraunhofer's works 30 years previously, L. P. Seidel (16), his mathematical helper, published in a very remarkable paper the opinion already advocated here that Fraunhofer tried to correct his objectives in quite a superior way. He not only did away with spherical aberration by fulfilling—in Seidel's system of notation—the equation

$$s_1 = 0,$$

but tried to remove (by trigonometrical calculations) the coma, fulfilling as well the equation

$$s_2 = 0.$$

When in 1887 at the hundredth anniversary of Fraunhofer's birthday Ernst Abbe tried to give his audience an idea of Fraunhofer's personality and achievements, he emphasised—as Faraday had done—the three following points of novelty in Fraunhofer's work:

1. He refined the possibilities of obtaining and testing ideal spherical surfaces by inventing grinding and polishing machines and methods of testing by means of Newton's rings.

2. He gave a much deeper foundation to calculation for optical purposes by exactly defining the refractive indices for certain well-chosen dark lines of the spectrum, by a scientific treatment of the luminosity of the spectrum, and by finding a successful way of dealing with the aberrations of oblique pencils.

3. He improved the homogeneity of the optical glass in specially large meltings (of about 4 cwt.) and gave an exactly defined condition to be fulfilled by glass pairs yielding objectives without secondary spectrum. He made as well the first trial melting of such a pair in order to show the way to real success in this direction.

I wish to tender my special thanks to Drs J. S. Anderson and F. Springer: the former prepared a translation when I was prevented by illness from submitting the paper in time for reading at the meeting, and has also revised the part of my paper which was written in English; the latter kindly made all the blocks for me at a time when I was unable to attend to this work.

REFERENCES.

(1) v. ROHR, M. "Zur Entwicklung des ausziehbaren Handfernrohrs." *Z. f. Instrk.* 45 (Aug. 1925) 378–83, 2 figs. [277.]*

(2) —— "Zur Kenntnis früher Formen des Erdfernrohrs." *Ibid.* 40 (Jan. 1920), 15–20, 12 figs. [278, 279, 280.]

(3) *A Companion to the Telescope.* London: S. Bagster (1811). Pp. 60. 8vo. [279.]

(4) v. ROHR, M. "Die Entwicklungsjahre der Kunst, optisches Glas zu schmelzen (Ein Versuch zusammenfassender Darstellung)." *Die Naturwissenschaften,* 12 (26th Sept. 1924) 781–97, 1 fig.; 13 (10th July, 1925) 619–22, 1 fig. [282.]

(5) —— "Pierre Louis Guinand geb. den 20 April 1748, gest. den 13 Februar 1824." *Z. f. Instrk.* 46 (Mar. 1926) 121–37; 46 (Apr. 1926) 189–97, 14 figs. [282, 285.]

(6) —— "Die Voigtländersche optische Werkstätte und ihre Umwelt. Ein Ausschnitt aus einer weitergreifenden Darstellung photographisch-optischer Betätigung." *Ibid.* 45 (Sept. 1925) 436–54; 45 (Oct. 1925) 470–83; 46 (Feb. 1926) 76–8. [282.]

(7) —— "Joseph Fraunhofer als der Schöpfer der deutschen Feinoptik." *Die Naturwissenschaften,* 14 (4th June, 1926) 539–52. [283, 284, 285.]

(8) —— "J. Fraunhofers Forschungen zur Glasbeschaffenheit und Farbenhebung sowie seine Leitung der Glashütte zu Benediktbeurn." *Z. f. Instrk.* 46 (June, 1926) 273–89, 2 figs. and 1 Table. [283, 284, 285, 290.]

(9) KITCHINER, W. *The Economy of the Eyes.* Part II. Of Telescopes, being the result of thirty years' experiments with fifty-one telescopes, of from one to nine inches diameter. London: Geo. B. Whittaker (1825). Pp. xi + 485. 8vo. (1 Plate and some figures, not numbered.) [289.]

(10) SEITZ, A. *Joseph Fraunhofer und sein optisches Institut.* Berlin: J. Springer (1926). Pp. iv + 118, 6 Tables. 8vo. [290.]

(11) PRECHTL, J. J. *Praktische Dioptrik als vollständige und gemeinfassliche Anleitung zur Verfertigung achromatischer Fernröhre.* Nach den neuesten Verbesserungen und Hülfsmitteln und eigenen Erfahrungen. Wien: J. G Heubner (1828). Pp. xii + 296. 4 Tables. 8vo. [291.]

(12) STAMPFER, S. "Methode, die Krümmungshalbmesser eines Objektivglases zu messen, angewendet auf die Untersuchung einiger Frauenhoferschen (!) Objektive." *Jahrb. d. polyt. Inst. Wien,* 13 (1828) 30–51, 1 fig. (Plate II). [291.]

(13) —— "Über die Theorie der achromatischen Objektive, besonders der Fraunhoferschen." *Ibid.* 13 (1828) 52–124, 2 figs. (Plate II). [291.]

(14) AIRY, G. B. "On the Principles and Construction of the Achromatic Eye-pieces of Telescopes, and on the Achromatism of Microscopes." *Camb. Phil. Trans.* 2 (1827) 227–52. "On the Spherical Aberration of the Eye-pieces of Telescopes." *Ibid.* 3 (1827) 1–64, 1 Table. [291.]

(15) CODDINGTON, H. *A Treatise on the Reflexion and Refraction of Light, being Part I of a System of Optics.* London: Simpkin and Marshall (1829). Pp. xx + 296 (2), 10 Tables. 8vo. [291.]

(16) SEIDEL, L. "Zur Dioptrik. Über die Entwicklung der Glieder 3ter Ordnung, welche den Weg einer ausserhalb der Ebene der Axe gelegenen Lichtstrahles durch ein System brechender Medien bestimmen." *Astr. Nachr.* 43 (1856) 289–304, 305–20, 321–32. [291.]

* The numbers in square brackets refer to the pages of the text where the references are mentioned.

DISCUSSION

Mr D. Baxandall (partly communicated): This paper is a most valuable contribution to the history of optical work during the latter part of the 18th and the earlier part of the 19th century. Prof. von Rohr never fails to indicate the sources of information from which he draws his conclusions, indicating also to what extent his conclusions are reliable, and often giving suggestions as to the kind of additional information which is required to increase the reliability of his conclusions.

The account of the advances made by the great Fraunhofer will always be a valuable record. Prof. von Rohr must be congratulated on the acquisition of much new information, and on its presentation to the Society in such a clear and interesting manner.

The value of actual early instruments as historical documents has interested me for many years, and I am glad that Prof. von Rohr has made such good use of the Ramsden and other telescopes which were available for measurement. In dealing with an achromatic object-glass in any instrument signed by Ramsden, however, it should not be forgotten that Ramsden married John Dollond's daughter, and that he received a part of the Dollond patent rights in achromatic telescopes. The object-glasses on some of the instruments signed by Ramsden were undoubtedly made by Peter Dollond. Kitchiner refers to one instance, and there are other known examples of this kind. The extent to which Ramsden depended on Dollond in this matter can perhaps be decided when measurements have been made of a considerable number of examples signed by Dollond and Ramsden respectively.

Dr Kitchiner was certainly the author of the book (Ref. (3)). It appeared also in 1815, still anonymously, slightly revised, and with some additions. The reference to Peter Dollond's remark on the scarcity of good glass and the cessation of the glass-house at Ratcliffe appears in both the above books, and also in Ref. (9). There was a glass-house at Ratcliffe as early as 1623, its owner being Abraham Bigoe. In *Environs of London*, vol. III, p. 473, published in 1795, Lysons refers to the industries of the hamlet of Ratcliff, including "Bowles's celebrated manufacture of window glass, established by the great-grandfather of the present proprietor, who is said to have been the first to manufacture crown glass in this kingdom," adding that "it has certainly been brought to its present improved state by his family." The cessation was presumably caused by the construction of the London Dock (1802–1805 and later). The notorious Ratcliff Highway was later re-named St George's Street, and the present Glasshouse Street, which runs from the east end of Royal Mint Street down to the London Docks, marks approximately the position of this once-famous glass-works.

According to Ree's *Cyclopaedia* 1819 (Art. "Glass") there were "formerly in London two kinds of crown glass, distinguished by the places where they were wrought; viz. *Ratcliff crown glass*, which is the best and clearest, and was first made at the Bear Garden, on the Bank-side, Southwark, but since at Ratcliff...and *Lambeth crown glass*, which is of a darker colour than the former, and more inclining to green...."

Mr J. Rheinberg exhibited an original set of Schwerd's diffraction discs and gratings, which he thought the members would be interested to see as they were constructed by Schwerd to demonstrate Fraunhofer's work on diffraction phenomena. He also exhibited a copy of Schwerd's book entitled *Die Beugungerscheinungen aus den Fundamentalgesetzen der Undulationstheorie analytisch entwickelt und in Bildern dargestellt*, which was published in Mannheim in 1835 and contains numerous illustrations partly hand-coloured.

Prof. von Rohr (communicated): I am very grateful for all the valuable additions made by Mr Baxandall—the mention of the second edition (1815) of Ref. (3), the statement of the close alliance between J. Ramsden and P. Dollond, and especially the detailed information about the glass-house at Ratcliffe and the Lambeth crown glass. I am sure that my colleagues in Germany will appreciate the value of this new information and join in my expression of gratitude.

DISCUSSION ON COLOUR TERMINOLOGY

THE following discussion took place on 14th April, 1926, during the Session of the Optical Convention, by permission of the Papers Committee of the Convention.

Introduction. By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S., Member of Council of Optical Society.

As a first step towards removing the confusion which at present exists in the use of colour terms by artists, physicists, psychologists, colour technicians, and others interested in colour, the Optical Society of America recently issued a questionnaire on colour terminology to elicit information as to the most popular usages of various words to denote the different attributes of colour. A few of these questionnaires were distributed in this country by British members of the Optical Society of America, and the replies were forwarded to that Society.

Subsequently our own Optical Society decided that further action in this country was desirable, and appointed a sub-committee, with Professor L. C. Martin as Secretary, to deal with the matter. This committee was supplied with additional copies of the American questionnaire and distributed these to various people known to be interested in the subject.

I have been asked by the committee to give an account of these preliminary activities as an introduction to the present discussion, which the committee hopes will be of value in assisting it in its further deliberations.

The questionnaire consisted of a series of six questions, all of which, excepting the first, were illustrated by coloured charts. Unfortunately it is not possible to reproduce the charts here*, and it will be necessary to attempt to convey in words the significance of the various questions. This naturally presents considerable difficulty in view of the fact that the whole discussion arises from the present lack of words having definite and generally accepted significance. It is hoped, however, that comparatively little ambiguity may be found in the descriptions given.

The questions were preceded by the note—

First. Please read all the questions. After each question is given a list of words in common use which may be used for answers. As the Committee desires to influence the selection in no way, the list is always presented in alphabetical order. Use any other word, or words, that you think more desirable. Do not use the same word to describe two different attributes or characteristics.

The first question, which is self-explanatory, was—

(A) What word or words would you use to designate that agent, force, or action in nature, by the operation of which on the organs of sight, objects are rendered visible or luminous?

Light
Luminous Energy
Radiant Energy
Radiation
Visible Radiant Energy
Visible Radiation

* The charts were, however, exhibited by means of an epidiascope at the meeting.

The second question was—

(B) Samples in Group (*a*), Chart I, differ from each other in only one characteristic; that is, they are either lighter or darker. Samples in Group (*b*), Chart I, differ from each other not only in light and dark, but each recalls to mind some part of the rainbow or pectrum. In other words, there is a fundamental difference between Groups (*a*) and (*b*).

What word or words would you use to designate the samples in Group (*a*)?

What word or words would you use to designate the samples in Group (*b*)?

Group (*a*)	Group (*b*)
Achromatic Colors	Chromatic Colors
Grays	Colors
Gray Colors	Hueful Colors
Gray Tones	Colored Tones
Neutral Colors	Colors

NOTE: Arranging in pairs is only for convenience. Use any combination.

The chart illustrating this question contains two groups of samples. Group (*a*) consists of nine squares, one of which is white, another black, while the remainder are of intermediate character, *all producible by mixing white and black pigments* in varying proportions. Group (*b*) consists of a miscellaneous set of eighteen samples; yellow, blue, green, red, purple, etc., the common characteristic being that *none of them can be produced by mixing white and black pigments only*.

The third question was—

(C) All the samples on Chart I, both Group (*a*) and Group (*b*), have been gathered together on Chart II.

What word or words would you use to designate all the samples on this chart?

Colors
Tones
Visual Sensations
Visual Tones

The chart illustrating this question contains thirty-six samples, including both groups of Chart I, arranged indiscriminately.

The fourth question was—

(D) Samples 1 to 9 on Chart III are arranged in a simple sequence. As the numbers increase the samples become lighter. This sequence represents the first attribute of the samples on Chart II, which, for convenience, we shall call Attribute *X*.

What word or words would you use to designate this sequence (Attribute *X*)?

Brightness
Brilliance
Intensity
Luminosity
Value

To illustrate this question, Chart III contains three sets of samples, arranged in separate rows, each row to be considered by itself. The attribute *X*, which it is required to name, varies from one end of each row to the other. The samples in each row are intended to be related to one another in such a way that a sample having a higher value of *X* could be made to look exactly like one having a lower value of *X* by merely reducing the illumination falling on the former without altering the illumination of the latter. Thus suppose we extract two samples from any one row such that the *X* of one is twice that of the other. If we so arrange

matters that instead of seeing both samples under the same degree of illumination, we can reduce the illumination on the former to a half of that on the latter, then the samples should look alike.

The fifth question was—

(E) In Chart IV the Groups (*c*) and (*d*) are similar in attribute *X*, but Group (*c*) is consistently different from Group (*d*) in a second attribute which we shall call *Y*. This attribute is shown in an orderly way in Chart V where the samples 0, 1, 3, 5, 7, 9 are arranged in a sequence (Attribute *Y*). As the numbers increase Attribute *Y* becomes more strong, as is shown from the sample in the center to the Red, Yellow, Green, Blue, and Purple.

What words or word would you use to designate Attribute *Y*?

Chroma
Color Intensity
Purity
Saturation
Strength

This question is best illustrated by Chart V, which contains five sets of samples. One of these sets has a red sample at one end and a gray sample at the other, the intermediate samples being such as an artist could produce by mixing gray pigment with red pigment in varying proportions. The red pigment alone gives a colour with the highest value of the attribute *Y*, which is diminished by admixture of gray pigment. The other series of this chart consist of gray and yellow and their intermediate mixtures; gray and green and their mixtures; gray and blue and their mixtures; gray and purple and their mixtures. Thus, whatever the nature of the colour concerned, *Y* is the attribute which is lowered by admixture with gray.

In attempting to describe the properties of this chart in terms of the pigments required to reproduce the various series, gray pigment has been mentioned. Gray pigment of course is simply a mixture of black and white pigments. It is the increase in "whiteness" which corresponds to the reduction in attribute *Y*, the black constituent of the gray pigment merely serving to counteract the increase in attribute *X* (see question (*D*)) which would in general accompany the admixture of white pigment with any other.

It may help to render clear the distinction between the two attributes *X* and *Y* in questions (D) and (E) if we assume that an artist has on his palette a certain pigment, or mixture of pigments, which when painted on his canvas produces a certain colour. If he mixes with the datum pigment some pure black pigment the resulting colour will be *reduced* in attribute *X* and will be unaffected in attribute *Y*. If, however, white instead of black be added to the datum pigment, there will in general be an increase in attribute *X*, but in addition a change in quality which is described by saying that the colour becomes paler. This change in quality is a diminution in attribute *Y*.

The sixth question was—

(F) The word commonly used to designate the quality difference in the rainbow or spectrum (Attribute *Z* on Chart VI) is Hue.

Do you agree to the use of this term? Have you any other word or words to describe, *i.e.* differentiate, the quality difference in the rainbow or spectrum?

Chart VI contains a series of samples having similar (high) values of the attribute *Y* and differing simply in being red, orange, yellow, green, blue, violet, or purple. Thus attribute *Z* is the property in virtue of which we would describe a colour as red (or reddish), green (or greenish), etc.

The nature of the questions set should be reasonably clear from the foregoing explanations. The American Optical Society has published a statistical report of all the replies received by their Committee* giving the number of votes cast for various words. The total number of replies received was in the neighbourhood of 650.

The complete analysis would take up too much space here, and we need only quote the predominating results:

Question	Term preferred	Votes for
(A)	Light	491
	Visible radiation	43
(B) (*a*)	Grays	207
	Neutral colors	108
	Achromatic colors	107
(B) (*b*)	Colors	282
	Chromatic colors	151
(C)	Colors	384
	Tones	82
(D)	Value	191
	Brightness	175
	Luminosity	52
(E)	Saturation	171
	Color intensity	147
	Purity	112
(F)	In favour of "Hue"	553
	Not in favour of "Hue"	34

The total number of replies received by the British Committee is only thirty at the time of writing. A statistical analysis of the preferences of such a small number is of little value, but for what it is worth it may be given here.

Question	Term preferred	Votes for
(A)	Light	24
	Visible radiation	3
(B) (*a*)	Grays	9
	Gray colours	5
	Neutral colours	5
	Neutral tones	5
(B) (*b*)	Colours	21
	Chromatic colours	3
(C)	Colours	18
	Tones	6
(D)	Intensity	12
	Luminosity	5
	Shade	4
	Brightness	3
(E)	Purity	12
	Colour intensity	4
	Saturation	3
(F)	In favour of "Hue"	25
	Not in favour of "Hue"	4

* Appendix to programme of Tenth Annual Meeting, October 1925.

To summarise these results we note that in reply to question (A) there is an enormous majority for *light*. For question (*B*) (*a*) *grays* has a two to one majority over any other term. There is a marked preference for *colours* without any adjective for both the questions (B) (*b*) and (C). There is also practical unanimity in favour of *hue* in reply to question (F).

As regards the foregoing questions the replies are adequate to give a fairly definite indication of popular preference. Moreover, British opinion, in so far as the small number of respondents may be taken as typical of those interested, is in very striking agreement with American opinion.

As regards question (D), the largest number of American votes go to *value*, with *brightness* a fairly close second. Unfortunately this question necessarily became ambiguous when illustrated by a coloured chart, and a study of the detailed analysis given in the American Report shows that this ambiguity is reflected in the replies.

The word *value*, as used in this connexion, is not really synonymous with the other words—brightness, luminosity, etc.—suggested for attribute *X*. These latter terms without qualifying adjectives are indicative of the *total amount of light* entering the eye from unit area of the observed surface. They depend both on the properties of the surface and on the intensity of the illumination incident on the surface.

For a given illumination, different objects look of different brightnesses owing to the fact that a greater or less proportion of the incident light is absorbed. The *fraction* of the incident light which reaches the eye after reflection (or transmission) by a coloured body is what is denoted by *value*. *Value* therefore does not mean the same thing as brightness but is the ratio of two brightnesses, namely the brightness of the reflected (or transmitted) light and the brightness of the incident light. *Value* is, therefore, synonymous with reflecting power, and is a property of the coloured material, whereas *brightness* (or *luminosity*) depends jointly on the reflectivity of the observed surface and the strength of the illumination.

For example, suppose we are in a room, illuminated by several similar lamps. The various objects in the room have various colours each of a certain *value*. If we turn off some of the lamps the *brightness* of all objects is reduced, but their *values* are not affected*. Now any attempt to give a pictorial illustration of variation of brightness can only be done by means of pigments of different reflecting powers, and the interpretation of question (D) will depend on the aspect of colour with which the person answering it is most familiar. Artists, printers, dyers, and people whose association with colour is mainly in connexion with coloured substances and their properties, will assume the question to refer to the variation in reflectivity of the pigments used to illustrate it, whereas physicists, physiologists, photometrists, and those whose interest is mainly in the attributes of visual sensation will assume the question to refer to differences in brightness.

This difference in interpretation is clearly revealed by an analysis of the voting.

* Assuming the brightness does not fall below that at which the properties of the eye are altered by the Purkinje effect.

Thus of the 191 votes for *value* 164 were from persons engaged in Art, Art-Education, and Industry, that is to say persons primarily concerned with pigments, inks, etc., while only 27 votes came from persons engaged in research, which presumably includes physicists, physiologists, psychologists, etc., who are concerned with visual science. On the other hand, of the 175 votes for *brightness*, 153 came from persons engaged on research.

It appears legitimate to regard these figures not as indicating a pronounced difference between the two classes as regards preference for a word to indicate the same attribute, but as indicating that they were, in fact, answering essentially different questions.

I interpret the voting on this question as indicating that *brightness* is the word predominantly favoured to denote the attribute of a visual sensation which can be altered by merely turning lamps off or on, while *value* is the word predominantly favoured to denote the fraction of the incident illumination which is reflected by an opaque object or transmitted by a filter.

The British returns are too few to provide any statistical evidence as to how the question was interpreted. The word *intensity*, a general quantitative term of which the significance is not restricted to visual phenomena at all, appears to have preference among those who actually returned papers.

As regards question (E), the American returns give a preference to *saturation* with *colour intensity* and *purity* not far behind. The British returns show a distinct preference for *purity*.

The foregoing summary includes the only information at present available as to the preponderating usages of words to denote the different attributes of colour. They are majority usages only, and of course give the same weight to the opinions of those who may have given little thought to the matter as to the views of those who have studied it carefully. It does not follow therefore that these preferences are the best suited to the needs of colorimetric science. They form, however, an excellent basis for discussion.

GENERAL DISCUSSION

Mr J. Guild: As regards most of the questions set in the questionnaire I am in agreement with the majority as revealed by the American report. For question (A) no other word appears to have a claim comparable with *light*. It is a short single word which by long popular usage denotes exactly what is intended. The habit among scientific writers of extending the meaning of *light* to include radiation generally, as when referring to *ultra-violet light*, *infra-red light*, etc., is to be deprecated. It destroys the special significance of the word and reduces it to a mere synonym for radiation or radiant energy. This impoverishes the language without serving a single useful purpose in compensation. The term light should be restricted to signify only such radiation as evokes the sensation of vision.

As regards questions (B) and (C), I am glad to note that by their replies the

majority reject the invitation to give *white* a special place in colour terminology. White, with its darker shades—grays—differs from all other colours, but so does yellow and its shades (the browns), green and its shades, blue and its shades, etc. It is just as easy (and logical) to divide visual sensations into, say, yellow and the rest as to divide them into white and the rest.

The quality in which white differs from red or green is the same as that in which they differ from one another. In terms of the trichromatic theory they differ in the relative stimulations of the three primary sensations. There is nothing fundamentally unique in the relative stimulations corresponding to white.

The tendency to regard white as something distinct from a colour—as including all colours, and so on—is merely an arbitrary way of regarding the matter, based on qualitative aspects of facts which loomed large in the early development of the science of light.

Because it was found that "white" light—by which was then meant anything from candle light to daylight—when split up by a prism, gave all the colours of the rainbow, white was regarded not as a colour but as a container of all colours. This property is not, of course, confined to white, but is possessed by any colour of which the energy distribution curve has not zero values within the visible spectrum, and does not serve to distinguish white from any of these colours—which are infinite in number.

The habit of regarding white as something other than a colour is always an acquired one. To a child, or to any person who has not been taught otherwise, white is as much a colour as red or blue.

The unique positions occupied by white in more modern colour theory are entirely matters of convenient definition. White, like any other colour, can be produced by an infinite number of different energy distributions. Its most common presentation to mankind is, however, as the colour of the most universal illuminant, viz. average daylight, which gives it a special *practical* importance and makes it convenient, though by no means *necessary*, to give it a special position in systems of colour specification. Thus in trichromatic systems we conventionally choose units such that equal quantities of the primaries make white. This is convenient, but there is nothing to make such choice essential.

Another method of measurement and specification is based on the fact that some colours can be matched by mixing suitable proportions of monochromatic light and white, while all others can be mixed with suitable monochromatic light to match white. The position occupied by white in the above system can be occupied by an infinite number of other colours, as a glance at the colour triangle will show. There is nothing to indicate white as the essential choice for the fixed constituent. As a matter of fact, while some colours are less suitable than white from practical considerations, inasmuch as a larger proportion of the colour field has to be expressed on the complementary basis, others are more suitable than white on this ground.

As particular cases of special interest, either the extreme red or violet of the spectrum, or any colour lying on the straight line joining these in the colour

triangle, if employed as the fixed colour, will enable all colours to be expressed on the direct system, eliminating the complementary region altogether.

In short, there is no justification for the separate classification of the colours in group (*a*), such phrases as "chromatic colours," "achromatic colours," etc. being mere absurdities. White is simply one of the possible colour sensations. Any of the circumstances which appear to give it special significance will be found on examination to be questions of definition and to have no basis in any essential difference between white and other colours—at any rate for cone vision, with which alone we are concerned in colorimetry.

Popular usage affords us no guidance here, as colour is regularly used in popular speech either to include or exclude white, depending on the context. Thus in reply to a question as to the colour of someone's dress, white would be given without any sense of verbal inaccuracy just as readily as pink or mauve or blue. Similarly red, white, and blue are referred to as the colours of the Union Jack. Numerous examples of circumstances in which white is grouped with other colours without any special significance being attached to it occur in common speech. On the other hand in words such as "colourless," "coloured," etc., the exclusion of white from the comity of colours is implied. Phrases such as "The colour has faded from this material," to indicate that the material has become white, are commonly used.

This double significance for the word colour is evidently consciously supported by the majority of the respondents, who have voted for "colour," without any qualifying adjective in both the full and restricted senses.

As a matter of fact for purposes of conversation and general descriptive writing it is highly convenient to employ the word in either sense as required, and I have never met a single case in which any ambiguity as to the meaning was not entirely precluded by the context. There is certainly no difficulty in preventing ambiguity if the slightest care is taken with the context, and, this being so, no good purpose would be served by attempting to restrict the significance of "colour" for descriptive purposes to either one of its present usages, as a certain degree of freedom of expression would be lost without any corresponding gain in clarity.

Nevertheless, we should avoid attaching a *scientific* significance to the word "colourless," which would convey the impression that there is something fundamentally different in the sensation of white from the sensations of other colours which would properly put it in a class by itself.

Whites of various luminosities should therefore be described generally as shades of white, just as we speak of shades of red, shades of yellow, etc., to indicate different luminosity levels of these colours. The darker shades of white may conveniently be given the special name "gray," just as we give the name "brown" to the darker shades of red or yellow.

As regards the precise significance of the word "colour," I use this to connote the resultant of all the elements which go to make up the visual sensation.

It is frequently convenient to employ a term which excludes the luminosity element and connotes merely those properties which determine the position of

the colour on a colour triangle, or its hue and saturation on the monochromatic system. For this I find the term "colour quality"—or simply "quality" if the context renders the adjective superfluous—as convenient as any.

Thus two sensations having the same hue and saturation, or represented by the same point in a colour triangle, but differing in luminosity, are regarded as different colours though of the same colour quality. This agrees with popular terminology in which yellow and brown, white and gray, etc., are regarded as different colours.

As regards the attribute of the sensation which may be varied by turning lamps on or off I agree that *brightness* when used with its proper meaning is exactly what is wanted. *Luminosity* is, however, much more widely used in scientific literature than the small support given to it in reply to the questionnaire would indicate. It has the advantage that it is never erroneously used to indicate saturation, as sometimes happens to *brightness* in common speech. Thus the words "*bright colours*" are often heard in circumstances where one knows that colours of high saturation are meant. For this reason I have myself tended to use *luminosity* more often than *brightness*, without, however, excluding the latter, which in its correct usage is synonymous with luminosity. Scientific writers should scrupulously avoid using *brightness* to connote any other quality instead of, or in addition to, luminosity.

As regards the other terms mentioned in question (D), *brilliance* is simply a variant of *brightness* and shares its advantages and disadvantages. *Intensity* is a word not confined at all to visual phenomena. It is a useful quantitative term of general applicability. Thus we speak of intensity of pain, intensity of X-radiation, intensity of warmth (in phrases such as "the heat was intense"), intensity of emotion, etc., etc. Intensity, alone, relates to nothing in particular. We could use *intensity of visual sensation* to denote brightness, but why use four words instead of one.

Value, as mentioned in the introductory statement, does not signify brightness, but denotes the reflectivity of an opaque body—or the transmission factor of a transparent filter. I have used the words *luminosity-fraction* or *fractional luminosity* for this ratio in some of my papers. These phrases have the advantage of being practically self-explanatory, which *value* is not. The principal objection to *value*, however, is that it attaches a specialised and restricted meaning to a word which in its proper significance is applicable to any variable quantity. Take, for example, sentences such as the following: "It is possible to specify any colour whatever in terms of three arbitrarily chosen primaries by an equation such as

$$C = \alpha R + \alpha' G + \alpha'' B,$$

provided one or more of the coefficients may have negative values; the actual values depending, of course, on the primaries chosen."... "In order that luminosity measurements by the flicker and continuous vision methods should agree, it is necessary that the field size and field brightness should have the values specified."... "We may regard the ratio of any given stimulus to the threshold stimulus as the sensation-value of the stimulus," etc. In these sentences, all of which are of a type frequently occurring in writings on colour, we have value used in its ordinary

and only legitimate sense. Confusion is certain to arise if value occurs sometimes in its general sense and sometimes in a specialised sense.

If phrases such as *fractional luminosity* are regarded as unduly cumbersome, the word *shade* could be given this quantitative significance without altering in any way its true meaning or its utility for general descriptive purposes. Thus instead of merely using *shade* as a qualitative term, as at present, we could, in quantitative work, refer to the fact that a coloured object reduces the luminosity of the incident light to 35 per cent., say, by stating that its shade is 35 per cent. I should like to put forward the definite suggestion that *shade* be used in this way.

As regards question (E), I agree with the American majority in preferring *saturation* to any of the suggested alternatives. For numerical purposes this has to be expressed as a percentage, pure spectral colours being arbitrarily regarded as having a saturation of 100 per cent. A mixture of a spectral colour with white, in which the spectral colour contributes x per cent. of the total brightness, would have a saturation of x. This is the sense in which I have hitherto used *saturation* myself.

While the spectral colours are the most saturated which the eye can see when functioning normally, it is nevertheless possible under special conditions of fatigue to perceive colour compared with which the ordinary appearance of the corresponding spectral colour is pale. Colour sensation is not therefore limited as regards saturation by that normally producible by objective stimuli, and may not even be limited to degrees of saturation producible by objective stimuli under any possible conditions of the eye. These distinctions are of importance in discussions of visual theory, and some writers have recently evinced a tendency to restrict the use of *saturation* to denote the subjective impression which increases as a colour becomes less pale, and vice versa, employing the alternative term *purity* for the objective proportion of spectral light in the mixture of spectral light and white which matches the colour.

My own view is that in the great bulk of the writings on colour only the objective ratio is in question at all, and *saturation*, where not otherwise specified, may conveniently be used with this quantitative significance. In those papers in which from the nature of the problems discussed ambiguity may arise, the difficulty can be very well met by using the terms *subjective saturation* and *objective saturation*. In any case I dislike the word *purity* so commonly used in this connexion. *Pure* and *purity*, like *intensity* and *value*, are words of general applicability which it is undesirable to limit in significance by special definitions. It is as frequently desirable to speak of a pure white as of a pure red or green. In fact it may be convenient to apply the term *pure* to any colour whatever, which may be of unique significance for the purpose of a particular problem, in order to denote its freedom from admixture with appreciable amounts of any other colour. It is surely a bad principle to give specialised and restricted meanings to widely used descriptive words, except in cases where the ordinary legitimate usage of the word has no application within the field of phenomena for which the specially defined term is required. Purity in its ordinary unrestricted sense has obvious applications in connexion with colour and it is undesirable to give it the very specialised quantitative significance

involved in using it to denote the proportion of monochromatic light in a mixture of this with white.

There appears to be practical unanimity in describing the attribute which varies progressively along the spectrum by the word *hue*. Although this is by no means the common usage of the term—any difference of colour, even when entirely a difference of saturation, or of shade, being popularly referred to as a change of hue—there would seem to be no more suitable term to denote the attribute in question.

We must define *hue*, therefore, as the attribute which varies with the predominant wave-length. All colours having the same predominant wave-length, that is to say, all colours which can be produced by mixing a given monochromatic radiation with various proportions of white, are regarded as having the same hue. In general all colours which lie on any straight line radiating from white in the colour triangle are of the same hue.

A few words must be said as regards the use of colour nomenclature to describe objects. The colour sensation produced by the light from any object depends not only on the properties of the object but on the energy distribution of the incident light. All difficulty is avoided, if in referring to the colours of objects we assume, unless the contrary is specified, that the object is illuminated by "white" light of standard energy distribution. Thus by describing a tablecloth as "white," a dress as "green," etc., we refer to the fact that the objects have these colours when illuminated by standard white light. We know that the tablecloth looks red in red light, blue in blue light, etc., but it is highly convenient to call it a white tablecloth all the time. We save an enormous amount of circumlocution by being able to make a statement such as "A purple tie is red in red light and blue in blue light." There is no contradiction in terms here, since the words "purple tie" denote an object having the perfectly definite property of reflecting purple light when illuminated by standard white light, and the subsequent remarks refer to effects exhibited by such an object under the special conditions specified.

Dr Herbert E. Ives: I wish to take this opportunity to express, on behalf of the Color Terminology Committee of the Optical Society of America, our appreciation of the co-operation on the part of the English Optical Society in securing expressions of opinion on this question of color terminology. It has been very fully brought out by Mr Guild that the terminology of color is excessively confused and that it is of real importance for the advancement of color science and for the education of users of color that some approach to a universal language of color be made. The first step in this direction has appeared to us to be the accumulation of information on existing color terms and information as to the extent to which recognised terms are actually used. When the complete results of the color terminology questionnaire are in, we shall be in a position, we hope, to make recommendations which can be adopted with the minimum of opposition.

The words to be used to designate various color qualities fall, from the standpoint of the present problem, under three categories. We have, first, those words

about which there is substantial agreement. These will present little difficulty. We have, second, words which have become standardised in certain allied branches of optical science and should, for that reason, be adopted if possible. One word in this category is, in my mind, the word *brightness*, which has been in use for many years in photometric nomenclature and should, for that reason, be adopted in color terminology. In the third category are the color qualities about which there is complete disagreement. In the case of these qualities we have the alternative of picking one of the disputed terms or of coining entirely new words. These will present the most difficult problem.

Prof. L. C. Martin: The agreement on questions (A), (B), (C), and (F) of the questionnaire seems definite enough, and it remains to discuss more thoroughly the more contentious questions (D) and (E). Whatever may be the outcome in regard to the word selected, I think it is sound to keep one word for one basic idea. If then, in photometry for example, it is desired to give a particular significance to the word, it might be done easily by an adjective; thus *photometric brightness* will have a definite significance in prescribing the kind of scale in which brightness is to be measured. On the other hand the psychologist may wish to discuss *sensation brightness*, and the physicist may attach a definite meaning to *colorimetric brightness*. The use of an adjective will always act as a danger signal, and is much better than the employment of different nouns, which have at the same time highly uncertain colloquial uses. Few of us are so concise and sententious as to be able to object to the extra word if by its use we render our meaning more exact.

It is of little use to expect that we can immediately reform the common uses of language. Scientific people must first set their own houses in order. The next people to approach are the teachers in our schools. Whether we shall ever manage to agree with the artists I have no idea. I hope that it will be possible for scientific societies like the Faraday, the Optical, and the Physical Societies to take up the question in conjunction with the more technical organisations. The interest shown in the present answers would certainly justify a joint discussion at some time in the near future.

THE STATIONARY VALUE OF AXIALLY SYMMETRIC FUNCTIONS. PART I

BY T. SMITH, M.A., F.INST.P.

(Optics Department, The National Physical Laboratory.)

MS. received, 27th May, 1926. Read, 10th June, 1926.

ABSTRACT. The formula for the stationary value of a function given in the paper "Lagrange's Theorem and Stationary Functions" is put into a form which shortens the calculations involved in applying it to functions which possess special kinds of symmetry, such as that corresponding to symmetry about an axis.

INTRODUCTION

IN a previous paper* the stationary value f_s of a function f of several variables $x_h, x_i, x_j, \ldots$ has been found in the form

$$\begin{aligned} f_s = f &+ \tfrac{1}{2}(0) + \tfrac{1}{2!}(1) + \tfrac{1}{3!}(2) + \tfrac{1}{2}(11) + \tfrac{1}{4!}(3) + \tfrac{1}{2!}(12) + \tfrac{1}{2}(111) \\ &+ \tfrac{1}{5!}(4) + \tfrac{1}{3!}(13) + \tfrac{1}{8}(22) + (112) + \tfrac{1}{2}(1111) \\ &+ \tfrac{1}{6!}(5) + \tfrac{1}{4!}(14) + \tfrac{1}{12}(23) + \tfrac{5}{12}(113) + \tfrac{5}{8}(122) + \tfrac{5}{3}(1112) + \tfrac{1}{2}(11111) \\ &+ \ldots \qquad \ldots\ldots(1), \end{aligned}$$

where the symbols consisting of numbers within brackets denote certain symmetrical sums, and x_h is everywhere given a special value y_h found from the leading terms of f. To a second approximation a correction h is found to this value of x_h, so that the value of x_h derived from the conditions $\dfrac{\partial f}{\partial x_h} = 0$ for all values of h is approximately $x_h = y_h + h$. When h appears as a suffix except in the form x_h or y_h it always denotes differentiation with respect to x_h. Then the meaning of the sums is apparent by quoting a typical term. Thus $(0) = hv_h$, $(1) = hi_hv_i$, and is also equal to hiv_{hi}, $(112) = hi_hj_ikl_{jk}v_l = hi_hj_iklv_{jkl}$, etc. Here v (which is always differentiated at least once) denotes the part of f not used in finding the y's, the number of factors in the term is greater by two than the sum of the numbers in the brackets, and these bracketed numbers denote the number of times the factors are differentiated, the first differentiation of v being neglected. Thus in the terms denoted by (112) two factors are each differentiated once and one is differentiated twice. The summation

* "Lagrange's Theorem and Stationary Functions," *Trans. Opt. Soc.* 27 (1925–26) 43.

is carried out by giving to each of the letters $h, i, j, \ldots$ every possible value, so that every letter but v is a dummy.

In many problems the variables x_h, x_i, x_j, ... always (apart from first order terms) enter into f in certain combinations, so that the number of variables is effectively reduced. A very frequent example arises when there is axial symmetry. For instance there may be four variables x_1, x_2, x_3, x_4, which always occur in the combinations

$$a = \tfrac{1}{2}(x_1^2 + x_2^2), \quad b = x_1x_3 + x_2x_4, \quad c = \tfrac{1}{2}(x_3^2 + x_4^2) \qquad \ldots\ldots(2).$$

There are then great advantages in regarding a, b, and c as the real variables, and it is desirable to express (1) in a form in which f_s is an explicit function of these variables alone. The problem is therefore the transformation of the various terms in (1) into a form in which the differentiations occur with respect to a, b, and c.

The Fundamental Equations

Any attempt to effect this transformation by direct substitution must be ruled out for all but a few of the simpler terms owing to the magnitude of the labour involved. The main object of this paper is to show how the transformation can be effected by a formal process to any desired order of accuracy. The explanation will be facilitated by the introduction of a special notation. The notation used in (1) is possible because the corrections h, i, j, ... are related to the differential coefficients of v in a way which allows suffixes to be interchanged. If another function is substituted for v this interchange is no longer valid as far as the new function is involved, though as all the other letters are dummies interchanges among them are permitted. We therefore use square brackets [ ] in place of the round brackets ( ), and insert an additional figure at the end which is less by unity than the number of differentiations suffered by the new function, say q. Thus the sum whose typical term is hq_h will be denoted by [00], the sum $hi_hj_iq_k$ by [1110], $hij_hk_iq_{jk}$ by [0111], $hijk_hq_{ijk}$ by [012]. The new notation requires one more figure to be inserted than in the old expressions, with the result that 0 is always present as one of the entries, and this may conveniently be the first letter if q carries more than one suffix, and must be the last if it carries but the one. Braces { } will be reserved for use in defining functions differentiated with respect to the new variables. Taking the variables (2) and assuming as the expression for f

$$f = f_0 + x_1\xi_1 + x_2\xi_2 + x_3\xi_3 + x_4\xi_4 + \alpha a + \beta b + \gamma c + v \qquad \ldots\ldots(3),$$

the function $\{pq\}$ is defined by the equation

$$\begin{aligned}\{pq\} = \frac{1}{2(\beta^2 - \alpha\gamma)}\{&(ap_a + bp_b + cp_c)(\gamma q_a - 2\beta q_b + \alpha q_c)\\ &+ (\gamma p_a - 2\beta p_b + \alpha p_c)(aq_a + bq_b + cq_c)\\ &+ (\alpha a + \beta b + \gamma c)(2p_bq_b - p_aq_c - p_cq_a)\} \qquad \ldots\ldots(4),\end{aligned}$$

where p and q are any functions of a, b and c. Since the expression is symmetrical in p and q, $\{pq\} = \{qp\}$, and as it is homogeneous of the first order in both variables, any constant factors may be placed outside the brackets.

The correcting terms to the first approximations for x_1, x_2, x_3 and x_4 are respectively

$$\left.\begin{aligned}
&\frac{1}{\beta^2-\alpha\gamma}\{x_1(\gamma v_a-\beta v_b)+x_3(\gamma v_b-\beta v_c)\}\\
&\frac{1}{\beta^2-\alpha\gamma}\{x_2(\gamma v_a-\beta v_b)+x_4(\gamma v_b-\beta v_c)\}\\
&\frac{1}{\beta^2-\alpha\gamma}\{x_1(\alpha v_b-\beta v_a)+x_3(\alpha v_c-\beta v_b)\}\\
\text{and}\quad&\frac{1}{\beta^2-\alpha\gamma}\{x_2(\alpha v_b-\beta v_a)+x_4(\alpha v_c-\beta v_b)\}
\end{aligned}\right\} \qquad \text{.....(5).}$$

It at once follows by direct summation that

$$[00]=hq_h=2\{vq\} \qquad \text{.....(6).}$$

Now if either of the factors in the product is differentiated with respect to a variable, like differentiations must be applied to the corresponding factor in the alternative expression. If q is replaced by v, this is equivalent to taking half the differential coefficient of the whole, so that

$$h_iv_h=\frac{\partial}{\partial x_i}\{vv\},$$

and hence

$$h1_h=\left(\gamma\frac{\partial}{\partial x_1}-\beta\frac{\partial}{\partial x_2}\right)\{vv\},$$

and so on. In other words, if $h\ldots_h$ is replaced by $hi_h\ldots_i$ in the expressions in braces, v is to be replaced by $\{vv\}$. We therefore have

$$hi_hq_i=[10]=2\{\{vv\}q\} \qquad \text{.....(7).}$$

The Higher Order Terms

It will now be shown that all the relations desired can be derived by the substitution of different expressions for q in (6) and (7). In the first place substitute iq_i for q in (6). Then

$$h(iq_i)_h=hi_hq_i+hiq_{hi}=[10]+[01]=4\{v\{vq\}\} \qquad \text{.....(8).}$$

If we substitute from (8) in (6) or from (6) in (8) we get

$$8\{v\{v\{vq\}\}\}=[110]+3[011]+[20]+[02] \qquad \text{.....(9),}$$

also from (7) in (6)

$$4\{v\{\{vv\}q\}\}=[110]+[011]+[20] \qquad \text{.....(10),}$$

and from (6) in (7)

$$4\{\{vv\}\{vq\}\}=[110]+[011] \qquad \text{.....(11).}$$

It will be noted that any substitution replaces the last figure in the square brackets by a series of combinations of figures distributed according to the multinomial theorem with index greater by unity than the final figure of the expression within which the substitution is made. The comparison of (8) and (11) illustrates this, as also the pairs (9) and (15), and (10) and (16).

By substituting from (8) in (8), or (6) in (9), or (9) in (6) we find

$$16\,\{v\,\{v\,\{v\,\{vq\}\}\}\} = [1110] + 7\,[0111] + 4\,[120] + 4\,[021] + 6\,[012] + [30] + [03] \quad \ldots\ldots(12),$$

and by other substitutions which are sufficiently obvious the further relations of this order

$$8\,\{v\,\{v\,\{\{vv\}\,q\}\}\} = [1110] + 3\,[0111] + 4\,[120] + 2\,[021] + [012] + [30] \quad \ldots\ldots(13),$$

$$8\,\{v\,\{\{vv\}\,\{vq\}\}\} = [1110] + 3\,[0111] + 2\,[120] + [021] + [012] \quad \ldots\ldots(14),$$

$$8\,\{\{vv\}\,\{v\,\{vq\}\}\} = [1110] + 3\,[0111] + [120] + [012] \quad \ldots\ldots(15),$$

and

$$4\,\{\{vv\}\,\{\{vv\}\,q\}\} = [1110] + [0111] + [120] \quad \ldots\ldots(16)$$

are obtained. The continuation of the process is a comparatively trivial matter. The independent relations of the next order, all obtainable by substituting from (6) in the last five equations and from (7) in the previous three are given below in tabular form.

	[11110]	[01111]	[1120]	[0121]	[0112]	[220]	[022]	[130]	[031]	[013]	[40]	[04]
$32\,\{v\,\{v\,\{v\,\{v\,\{vq\}\}\}\}\}$	1	15	11	30	25	4	10	7	5	10	1	1
$16\,\{v\,\{v\,\{v\,\{\{vv\}\,q\}\}\}\}$	1	7	11	16	6	4	3	7	3	1	1	.
$16\,\{v\,\{v\,\{\{vv\}\,\{vq\}\}\}\}$	1	7	7	11	6	2	2	2	1	1	.	.
$16\,\{v\,\{\{vv\}\,\{v\,\{vq\}\}\}\}$	1	7	5	7	6	1	1	1	.	1	.	.
$16\,\{\{vv\}\,\{v\,\{v\,\{vq\}\}\}\}$	1	7	4	4	6	.	.	1	.	1	.	.
$8\,\{v\,\{\{vv\}\,\{\{vv\}\,q\}\}\}$	1	3	5	3	1	1	.	1	.	.	.	.
$8\,\{\{vv\}\,\{v\,\{\{vv\}\,q\}\}\}$	1	3	4	2	1	.	.	1	.	.	.	.
$8\,\{\{vv\}\,\{\{vv\}\,\{vq\}\}\}$	1	3	2	1	1	.	.	.	.	.	.	.

.....(17).

If the calculation is only to go one stage farther, the work can be simplified by returning to the old notation in the process. It has merely to be observed that when q is replaced by v the figure 0 is to be omitted from the brackets, the figures rearranged in order, and the square replaced by round brackets. When this is done several of the equations already found are seen to be redundant. The relations of the next order are given below in tabular form.

	(11111)	(1112)	(122)	(113)	(23)	(14)	(5)
$32\,\{v\,\{v\,\{v\,\{v\,\{vv\}\}\}\}\}$	16	131	94	77	25	16	1
$16\,\{v\,\{v\,\{\{vv\}\,\{v\,\{vv\}\}\}\}\}$	8	43	22	13	3	1	.
$16\,\{v\,\{\{vv\}\,\{v\,\{v\,\{vv\}\}\}\}\}$	8	37	14	11	1	1	.
$16\,\{\{vv\}\,\{v\,\{v\,\{v\,\{vv\}\}\}\}\}$	8	33	7	11	.	1	.
$8\,\{v\,\{v\,\{\{vv\}\,\{vv\}\}\}\}$	4	26	16	10	3	1	.
$8\,\{\{vv\}\,\{\{vv\}\,\{v\,\{vv\}\}\}\}$	4	9	1	1	.	.	.
$4\,\{v\,\{\{vv\}\,\{\{vv\}\,\{vv\}\}\}\}$	2	7	2	1	.	.	.
$4\,\{\{vv\}\,\{v\,\{\{vv\}\,\{vv\}\}\}\}$	2	6	1	1	.	.	.

.....(18).

On the left the eight functions are expressible in terms of only seven sums, and the eight must therefore satisfy an identity among themselves. It is easily seen that the equation

$$\{v\,\{v\,\{\{vv\}\,\{v\,\{vv\}\}\}\}\} + \{\{vv\}\,\{\{vv\}\,\{v\,\{vv\}\}\}\} + 2\,\{\{vv\}\,\{v\,\{v\,\{v\,\{vv\}\}\}\}\} + 2\,\{v\,\{\{vv\}\,\{\{vv\}\,\{vv\}\}\}\} = 3\,\{v\,\{\{vv\}\,\{v\,\{v\,\{vv\}\}\}\}\} + 3\,\{\{vv\}\,\{v\,\{\{vv\}\,\{v\,\{vv\}\}\}\}\} \quad \ldots\ldots(19)$$

is satisfied. In consequence of this identity the terms of this order in the expression for f_s may take different forms. To derive the formula required it is only necessary to solve the foregoing equations for the quantities in round brackets and substitute in (1). The result is

$$\begin{aligned} f_s = f &+ Dv + D^2v + \tfrac{1}{3}EDv + \tfrac{8}{3}D^3v \\ &+ \tfrac{1}{2}ED^2v + \tfrac{1}{6}DEDv + \tfrac{1}{3}D^4v \\ &+ \tfrac{8}{15}E^2Dv + \tfrac{1}{15}ED^3v + \tfrac{4}{15}D^2EDv + \tfrac{8}{15}D^5v \\ &+ \tfrac{1}{6}E^2D^2v + \tfrac{43}{15}EDEDv - \tfrac{17}{9}ED^4v - \tfrac{84}{15}DE^2Dv \\ &\qquad + \tfrac{1831}{450}DED^3v - \tfrac{149}{45}D^3EDv + \tfrac{7}{45}D^6v \\ &+ \ldots \end{aligned} \qquad \ldots\ldots(20)$$

where the operators D and E are defined by

$$Dq = \{vq\},\ Eq = \{uq\},\ u = Dv \qquad \ldots\ldots(21).$$

The term of the last order most conveniently omitted from f_s is D^2ED^2v, for it will be observed that the quantity $\{v\,\{\{vv\}\,\{v\,\{vv\}\}\}\}$ or DED^2v does not enter into the formula, its coefficient being zero. The sum of the coefficients of all the terms of any order is unity*. This follows from the fact that in (1) the coefficient of all the brackets enclosing only a series of 1's is $\frac{1}{2}$.

The derivation of this formula has followed from the consideration of a special substitution, viz. (2). From the method of development it is apparent that it will hold for all applications in which two relations such as (6) and (7) can be established, where $\{pq\}$ denotes a symmetrical function of p and q linear in each of these two functions.

* It may be remarked that the sums of the coefficients of the terms of the same order in (1) form the series

$$\frac{1^{-1}}{1!}, \frac{2^0}{2!}, \frac{3^1}{3!}, \frac{4^2}{4!}, \frac{5^3}{5!}, \frac{6^4}{6!}, \frac{7^5}{7!}, \ldots.$$

THE TREATMENT OF REFLECTION AS A SPECIAL CASE OF REFRACTION

By T. SMITH, M.A., F.Inst.P.

(Optics Department, The National Physical Laboratory.)

MS. received, 28th July, 1926. *Read (in abstract), 24th June*, 1926.

ABSTRACT. It is well known that reflection may be treated as a special case of refraction by assigning negative values to the refractive indices. The equation of reflection is frequently expressed incorrectly in optical treatises, and this erroneous relation immediately suggests the use of negative indices. The justification of the principle is considered in detail, and the subsidiary questions of sign in mixed reflecting and refracting systems are discussed. This leads to the computation of such systems on the supposition that the powers and reduced separations are independent of the direction in which the light is travelling at any stage of its passage through the system.

In books on geometrical optics a very usual plan is to consider reflection in the first place as the simpler problem, and afterwards to discuss the more complex events found when light passes from one body to another. This course has distinct advantages when the elementary properties alone are investigated, but the duplication of the onerous enquiries involved in advanced work should if possible be avoided. For such purposes it is of undeniable importance to be able to include the simpler problem of reflection in the general investigations necessary for refracting systems. That it should be feasible to do so is to be expected in view of the fact that the stationary character of the value for the path is a necessary and sufficient condition for the solution of either problem.

The method by which this aim may be satisfied is well known. Reflection is equivalent to a change in the sign of the refractive index. With this statement the subject is usually dismissed. That much more requires to be said becomes apparent to the student who considers any system other than a simple mirror. Such questions as "are the distances between the surfaces to be considered negative for the reflected ray?" and "are the curvatures of the surfaces met by the reflected light to receive the same sign as they bear for the original incident light?" arise at once, and, owing to the way in which reflection is traditionally treated, the answers are not found as readily as might be anticipated.

As has been pointed out before, the signs given to the angles in reflection are unsystematic. The angle of incidence is measured in one direction, the angle of reflection in another. This change in convention renders the assimilation of reflection to refraction unnecessarily difficult, and is probably responsible for the

inaccuracy in the analytical equations for reflection often found in ambitious treatises on optics. Since it is a fundamental principle in mathematical investigations to observe the same conventions consistently throughout an investigation, it is necessary to discard the usual statement of the law of reflection. The angles of incidence and of reflection must be measured from the same normal direction as the zero. Thus instead of the usual statement $\theta = \psi$ (Fig. 1), we must write

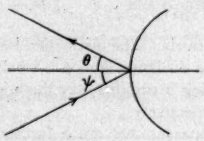

Fig. 1

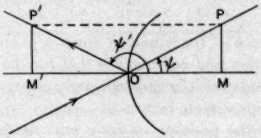

Fig. 2

$\psi + \psi' = \pi$ (Fig. 2) as the angular law of reflection. Disregard of this elementary principle has led some writers to give the law in the analytical form

$$\frac{L' + L}{l} = \frac{M' + M}{m} = \frac{N' + N}{n} \qquad \ldots\ldots(1),$$

while others, equally numerous, give

$$\frac{L' - L}{l} = \frac{M' - M}{m} = \frac{N' - N}{n}.$$

Obviously one or the other is wrong.

To decide which of these forms is correct, let OP and OP' be unit vectors drawn from the point of reflection O along the incident and reflected rays. Then if PM and $P'M'$ are perpendiculars to the normal, $OM = \cos\psi$, $OM' = \cos\psi'$, so that $P'P = \cos\psi - \cos\psi'$. By projecting the triangle OPP' on each of the coordinate axes in turn we find, since $P'P$ lies in the direction (l, m, n) of the normal, and (L, M, N) and (L', M', N') determine the direction in which the light travels,

$$\frac{L' - L}{l} = \frac{M' - M}{m} = \frac{N' - N}{n} = \cos\psi' - \cos\psi \qquad \ldots\ldots(2).$$

This may be compared with the law of refraction

$$\frac{\mu'L' - \mu L}{l} = \frac{\mu'M' - \mu M}{m} = \frac{\mu'N' - \mu N}{n} = \mu'\cos\psi' - \mu\cos\psi \ldots\ldots(3).$$

Direct comparison does not suggest that we should put $\mu' + \mu = 0$. Indeed the justification for the latter substitution is often based on the comparison of (3) with (1), an equation which is definitely incorrect.

Since $\psi + \psi' = \pi$, $\cos\psi' + \cos\psi = 0$, and the last expression in (3) takes the form $-(\mu' + \mu)\cos\psi$; when ψ is small this becomes sensibly equal to $-(\mu' + \mu)$.

In problems on refraction ψ and ψ' vanish together, and the corresponding quantity tends to the limit $(\mu' - \mu)$. It is by this comparison that the procedure of changing signs in reflection must be justified.

Since for refraction $\mu' \sin \psi' = \mu \sin \psi$, and for reflection

$$\sin \psi' = \sin (\mu - \psi) = \sin \psi,$$

in (2) and (3) we may substitute

$$\{2 - 2\cos(\psi' - \psi)\}^{\frac{1}{2}} \text{ and } \{\mu'^2 - 2\mu'\mu\cos(\psi' - \psi) + \mu^2\}^{\frac{1}{2}}$$

for $\cos \psi' - \cos \psi$ and $\mu' \cos \psi' - \mu \cos \psi$ respectively. The distinction between refraction and reflection is that for the former the deviation $\psi' - \psi$ is small, and for the latter it is approximately equal to π, showing in another way the origin of the approximate factors $\mu' - \mu$ and $-(\mu' + \mu)$ respectively.

In the position we have now reached it is apparent that much further consideration is needed before we may change the sign of the refractive index when light is reflected, for the change occurs only in one factor in an approximate expression. To deal with the matter further we require to take the geometrical relations into consideration.

Just as the positive directions for coordinates should be invariable, so the signs given to the curvatures of the surfaces and to the separations between the surfaces should be invariable. Thus if ξ, η, ζ is the point of refraction on a sphere of radius r, we shall always have, with the usual curvature sign convention,

$$\xi + lr - a = \eta + mr - b = \zeta + nr - c = 0 \qquad \ldots\ldots(4),$$

where (a, b, c) is the centre of the sphere. Equations (2) and (3) may therefore take the form

$$\frac{L' - L}{a - \xi} = \frac{M' - M}{b - \eta} = \frac{N' - N}{c - \zeta} = \kappa \qquad \ldots\ldots(5),$$

and

$$\frac{\mu' L' - \mu L}{a - \xi} = \frac{\mu' M' - \mu M}{b - \eta} = \frac{\mu' N' - \mu N}{c - \zeta} = K \qquad \ldots\ldots(6),$$

where $\kappa = \dfrac{\cos \psi' - \cos \psi}{r}$ and $K = \dfrac{\mu' \cos \psi' - \mu \cos \psi}{r}$.

Also if (x, y, z) is a point on the incident ray distant ρ from (ξ, η, ζ) in the direction in which the incident light travels, and (x', y', z') is similarly distant ρ' from (ξ, η, ζ) in the direction in which the emergent light travels, the relations

$$\frac{x - \xi}{L} = \frac{y - \eta}{M} = \frac{z - \zeta}{N} = \rho \qquad \ldots\ldots(7),$$

and

$$\frac{x' - \xi}{L'} = \frac{y' - \eta}{M'} = \frac{z' - \zeta}{N'} = \rho' \qquad \ldots\ldots(8)$$

are invariably true. Now if the light is reversed, the direction cosines are changed in sign but the media are encountered in the reverse order, so that K and the numerators in (6) are unaffected. The denominators, being the coordinates of definite points, are of course unchanged. Similarly in (7) and (8), $(x - \xi)$, etc.,

since they denote fixed distances, are not liable to change. The change in sign of the direction cosines is compensated by the alteration in the positive direction for the measurement of ρ and ρ'. Thus in refraction changes in direction may be neglected. Since the direction of the light is quite immaterial in refraction, and a change in the sign of the refractive indices throughout does not alter the equations, we may seek an analogy between reflection and a refracted ray traced backwards by replacing the vector ρ' by $-\rho''$, and consequently replacing L', M', N' by $-L''$, $-M''$, $-N''$. The equation of reflection now becomes

$$\frac{-L''-L}{a-\xi}=\frac{-M''-M}{b-\eta}=\frac{-N''-N}{c-\zeta}=\kappa=\frac{-\cos(\pi+\psi')-\cos\psi}{r} \quad \ldots(9).$$

The direction cosines L'', M'', N'' may be regarded as those of a real ray direction when refraction takes place after reflection. The vector ρ'' is measured in the ordinary direction for the refracted ray and the only change involved arises from the fact that on transferring the reference point from which these distances are measured from one surface to another the distances are increased where formerly they were diminished. In other words the separations become negative. There is however the further fact to be considered that these distances always occur in optical relations in association with an odd power of the refractive index, and since this has changed its sign, the reduced separations of the surfaces are not affected. The only case in which the reversal of sign has to be borne in mind is when repeated reflection takes place; the refractive indices will be alternately negative and positive for these reflections. Alternatively the signs of the powers on reflection may be determined by considering on which side of the surface the light falls.

Since optical formulas necessarily give reduced, and not real distances, the change in the direction of the vector coupled with a change in the refractive index may be neglected in finding the positions of the cardinal points of any system in air.

The conclusion we reach is thus that if we consider the powers of the surfaces unaffected by the reflection of light, and if their reduced separations are always considered positive, we may apply the ordinary refraction formula freely to reflecting or mixed reflecting and refracting systems.

As an example consider a lens consisting of a front refracting surface of power κ distant t from a back reflecting surface of curvature S. Let the refractive index be μ. Then by Cotes' theorem the power of the system written at length is

$$K=\kappa-2\mu S+\kappa-\frac{t}{\mu}\kappa(-2\mu S)-\frac{t}{\mu}(-2\mu S)\kappa-2\frac{t}{\mu}\kappa^2+\left(\frac{t}{\mu}\right)^2\kappa^2(-2\mu S)$$

$$=2\left(1-\frac{t}{\mu}\kappa\right)(\kappa-\mu S+t\kappa S),$$

and parallel incident light is brought to a focus in air at a distance d from the front surface where

$$dK=1-\frac{t}{\mu}(-2\mu S)-2\frac{t}{\mu}\kappa+\left(\frac{t}{\mu}\right)^2\kappa(-2\mu S)=1-2\frac{t}{\mu}(\kappa-\mu S+t\kappa S).$$

For light reflected at each surface in turn the corresponding expressions are

$$K' = \kappa - 2\mu S + 2\mu R + \kappa' - \frac{t}{\mu}\kappa(-2\mu S) - \frac{t}{\mu}(-2\mu S)(2\mu R)$$
$$- \frac{t}{\mu}(2\mu R)\kappa' - \frac{2t}{\mu}\kappa(2\mu R) - \frac{2t}{\mu}(-2\mu S)\kappa' - \frac{3t}{\mu}\kappa\kappa'$$
$$+ \frac{t^2}{\mu^2}\kappa(-2\mu S)(2\mu R) + \frac{t^2}{\mu^2}(-2\mu S)(2\mu R)\kappa'$$
$$+ \frac{2t^2}{\mu^2}\kappa(-2\mu S)\kappa' + \frac{2t^2}{\mu^2}\kappa(2\mu R)\kappa' - \frac{t^3}{\mu^3}\kappa(-2\mu S)(2\mu R)\kappa',$$

and $d'K' = 1 - \frac{t}{\mu}(2\mu R) - \frac{2t}{\mu}(-2\mu S) - \frac{3t}{\mu}\kappa + \frac{t^2}{\mu^2}(-2\mu S)(2\mu R)$
$$+ \frac{2t^2}{\mu^2}\kappa(-2\mu S) + \frac{2t^2}{\mu^2}\kappa(2\mu R) - \frac{t^3}{\mu^3}\kappa(-2\mu S)(2\mu R),$$

where $\kappa = (\mu - 1)R$, $\kappa' = (1 - \mu)S$.

These may be reduced to

$$K' = 2(R - S) + 4tRS + \{3 - 4t(R - S + tRS)\}\left(\kappa + \kappa' - \frac{t}{\mu}\kappa\kappa'\right),$$

and $d'K' = -2(1 - 2tR) + \{3 - 4t(R - S + tRS)\}\left(1 - \frac{t}{\mu}\kappa\right)$

respectively. The expression corresponding to the latter for light passing through the lens in the reverse direction and suffering two reflections on the way is obviously to be found by taking the coefficient of κ in the latter part of the expression for K' and the coefficient of *R with the sign reversed* in the remainder.

ON THE LIGHT TRANSMITTED AND REFLECTED BY A PILE OF PLATES

By T. SMITH, M.A., F.Inst.P.

(Optics Department, The National Physical Laboratory.)

MS. received, 20th July, 1926. *Read (in abstract) and discussed, 24th June*, 1926.

ABSTRACT. The properties of a series of media or of a pile of plates which absorb and scatter light, and the interfaces of which may also absorb and scatter as well as transmit and reflect light, are investigated without assuming that any of these factors are independent of the direction in which the light travels. It is shown that in general

(*a*) the ratio of the transmissive factors of a pile of plates in the two directions is independent of all reflective properties of the surfaces and of the order in which the plates are arranged,

(*b*) the transmissive factors in the two directions are equal if the individual transmissive and absorptive factors are the same for the two beams,

(*c*) the ratios of the light transmitted without reflection to the total light transmitted are equal in both directions,

(*d*) the reflective properties of the pile depend upon all the factors of the system, and the ratio of the two reflective factors is not independent of the order in which the plates are placed,

and in non-absorbing systems

(*e*) the sum of the intensities of the reflected and transmitted beams is equal to the intensity of the incident beam, and the ratio of the intensity of the reflected to that of the transmitted beam is equal to the sum of the corresponding ratios for the component plates or surfaces of the pile.

Such a pile evidently forms an exception to the rule (*d*) that the reflective coefficients depend on the order in which the plates are arranged.

Formulas for computing the transmissive and reflective factors for the pile are given.

The General Theory

Fresnel* and Stokes† have investigated the amount of light transmitted and reflected by a pile of similar plates. Recently Baker‡ has derived one of the formulas by an argument essentially the same as that used by both these authors, in order to point out some disadvantages of periscopes containing large numbers of lenses. The method consists in considering all the paths which the light may take, assigning the appropriate intensity to each beam, and summing all the emergent intensities. The following procedure avoids the summations and deals with a more general problem.

* A. Fresnel, *Œuvres Complètes*, vol. 2, p. 789.

† G. G. Stokes, *Proc. Roy. Soc.* 11 (1860–62) 545; *Collected Papers*, vol. IV, p. 145.

‡ T. Y. Baker, *Trans. Opt. Soc.* 22 (1920–21) 88.

The plates constituting the pile will not be specified initially, but the properties they must possess for the theory to apply to them will be inferred in the course of the investigation. The light encounters a series of surfaces at which it suffers reflection, refraction, and absorption, and is further subject to absorption in the substance of any plate between its bounding surfaces. Light may also be scattered, but the properties of the plates in this respect need not be specified separately for the purpose of this investigation, since in so far as the scattered light fails to contribute to the measured external beams it is immaterial whether it be considered to be absorbed rather than scattered, and in so far as it does contribute to these beams it may be taken into account by a suitable definition of the reflective and transmissive factors of the surfaces. This is permissible since the amount of scattered light is proportional to the intensity of the total light traversing the plate. The initial problem is the determination of the relative intensities of the incident, reflected, and transmitted beams in terms of the reflective, transmissive, and absorptive factors of the several surfaces and media of the pile.

Suppose that in the pile there are $n + 1$ surfaces. The intensities of the light on the two sides of any surface will in general differ, and it is convenient, in order to specify precisely the position to which any measures of intensity refer, to assign two reference numbers to each surface, one relating to each side of the surface. The integers 0 to $2n + 1$ are used for this purpose and are applied to the sides as they are met in order by light which traverses the whole pile without suffering reflection. Thus the mth surface has its sides numbered $2(m - 1)$ and $2m - 1$, and the medium between the mth and $(m + 1)$th surfaces stretches from the side $2m - 1$ to the side $2m$.

Denote the intensity of light travelling in the direction of m increasing by I and that of light travelling in the direction of m decreasing by J. Each of these letters may bear as suffix the number of the side which indicates where it is to be measured. Let the transmissive and reflective factors for light travelling in the direction of m increasing which is about to be resolved at the $(m + 1)$th surface be τ_{2m} and ρ_{2m} respectively. Similarly denote by τ_{2m+1} and ρ_{2m+1} the corresponding factors for light travelling in the direction of m decreasing about to impinge on the same surface. It is not to be assumed that the sum of τ_m and ρ_m is unity, since some light may be absorbed or scattered at the surface. The relations connecting the light intensities on the two sides of the surface are evidently

$$\left.\begin{aligned} J_{2m} &= I_{2m}\rho_{2m} + J_{2m+1}\tau_{2m+1} \\ I_{2m+1} &= I_{2m}\tau_{2m} + J_{2m+1}\rho_{2m+1} \end{aligned}\right\} \qquad \text{.....(1).}$$

Let the absorptive factors for light travelling across the mth plate in the directions m increasing and m decreasing be η_{2m} and η_{2m-1} respectively. Then

$$\left.\begin{aligned} I_{2m} &= \eta_{2m}\, I_{2m-1} \\ \text{and} \qquad J_{2m-1} &= \eta_{2m-1} J_{2m} \end{aligned}\right\} \qquad \text{.....(2).}$$

It is convenient to introduce new constants t and r defined by

$$t_\lambda = \tau_\lambda \eta_\lambda, \quad r_\lambda = \rho_\lambda \eta_\lambda \qquad \text{.....(3),}$$

and to replace equations (1) and (2) by

$$\left.\begin{aligned} J_{2m} &= I_{2m-1}r_{2m} + J_{2m+2}t_{2m+1} \\ I_{2m+1} &= I_{2m-1}t_{2m} + J_{2m+2}r_{2m+1} \end{aligned}\right\} \qquad \text{.....(4).}$$

These equations involve only I's with odd and J's with even suffixes, and to make them suffice for the complete solution of the problem it is only necessary to note that $\eta_0 = \eta_{2n+1} = 1$, and that the intensities I and J of the incident beams of light on the external faces of the pile appear as I_{-1} and J_{2n+2} respectively. Since these equations involve only the constants t and r, no distinction need be made between absorption within the medium and absorption at its surface. In using a train of equations like (4) for the investigation of the problem, either all the measurements must be made at the same point of each surface—implying normal incidence—or the intensity of the light incident upon the surface must not vary from point to point. This, together with the implied absence of interference effects, determines the only limitations which appear to be involved in the theory.

The relations (4) may be expressed in tabular form as follows:

J_0	I	J_2	I_1	J_4	I_3	J_{2n}	I_{2n-1}	J	I_{2n+1}
-1	r_0	t_1							
	t_0	r_1	-1						
		-1	r_2	t_3					
			t_2	r_3	-1				
.....									
						-1	r_{2n}	t_{2n+1}	
							t_{2n}	r_{2n+1}	-1

Denote the determinant obtained by taking the part of the above table having r_h and r_k as its extreme principal diagonal elements by $\Delta_{h,k}$. Then the Δ's are calculable from the difference equations

$$\left.\begin{aligned} \Delta_{2m,k} &= \Delta_{2m+1,k}r_{2m} - \Delta_{2m+2,k}t_{2m}t_{2m+1} \\ \Delta_{2m-1,k} &= \Delta_{2m,k}r_{2m-1} - \Delta_{2m+1,k} \end{aligned}\right\} \qquad \text{.....(5),}$$

or from

$$\left.\begin{aligned} \Delta_{h,2m} &= \Delta_{h,2m-1}r_{2m} - \Delta_{h,2m-2} \\ \Delta_{h,2m+1} &= \Delta_{h,2m}r_{2m+1} - \Delta_{h,2m-1}t_{2m}t_{2m+1} \end{aligned}\right\} \qquad \text{.....(6).}$$

Then if

$$T_{h,k} = (-t_h)(-t_{h+2})\dots(-t_k) \qquad \text{.....(7),}$$

h and k being both odd or both even, J_0 and I_{2n+1} are given in terms of I and J by

$$IJ\Delta_{0,2n+1} - II_{2n+1}\Delta_{0,2n} - JJ_0\Delta_{1,2n+1} + J_0I_{2n+1}\Delta_{1,2n} = 0 \qquad \text{.....(8),}$$

$$J\Delta_{1,2n+1} - I_{2n+1}\Delta_{1,2n} - IT_{0,2n} = 0 \qquad \text{.....(9),}$$

$$I\Delta_{0,2n} - J_0\Delta_{1,2n} - JT_{1,2n+1} = 0 \qquad \text{.....(10),}$$

$$J\Delta_{0,2n+1} - I_{2n+1}\Delta_{0,2n} - J_0T_{0,2n} = 0 \qquad \text{.....(11),}$$

$$I\Delta_{0,2n+1} - J_0\Delta_{1,2n+1} - I_{2n+1}T_{1,2n+1} = 0 \qquad \text{.....(12).}$$

These equations of course are not all independent. Equation (8) follows from (9) and (11) or from (10) and (12) by eliminating T_0 or T_{2n+1}. There is also an identical relation satisfied by the Δ's and T's, viz.

$$\Delta_{0,2n}\cdot\Delta_{1,2n+1} - \Delta_{0,2n+1}\cdot\Delta_{1,2n} = T_{0,2n}\cdot T_{1,2n+1} \qquad \text{.....(13),}$$

as may be seen by eliminating J_0 and I_{2n+1} from three of the four equations (9) to (12). The complete solution of the problem is thus contained in equations (9) and (10). Now I and J are arbitrary and independent, so that one or other of them may be made zero. It at once follows that for light incident on the first plate the transmissive factor of the pile is

$$\frac{I_{2n+1}}{I} = \frac{-T_{0,2n}}{\Delta_{1,2n}} \qquad \ldots\ldots(14),$$

and the reflective factor is

$$\frac{J_0}{I} = \frac{\Delta_{0,2n}}{\Delta_{1,2n}} \qquad \ldots\ldots(15).$$

For light incident on the last surface the corresponding factors are, for transmission,

$$\frac{J_0}{J} = \frac{-T_{1,2n+1}}{\Delta_{1,2n}} \qquad \ldots\ldots(16),$$

and for reflection

$$\frac{I_{2n+1}}{J} = \frac{\Delta_{1,2n+1}}{\Delta_{1,2n}} \qquad \ldots\ldots(17).$$

From the form of these ratios a number of conclusions may be drawn:

(*a*) The ratio of the transmissive factors of a pile of plates in the two directions is independent of all reflective properties of the surfaces and of the order in which the plates are arranged.

(*b*) The transmissive factors in the two directions are equal if the individual transmissive and absorptive factors are the same for the two beams.

(*c*) The ratio of the light transmitted without reflection to the total light transmitted is equal to $\Delta_{1,2n}$ in both directions.

(*d*) In general the reflective properties of the pile depend upon all the factors of the system, and the ratio of the two reflective factors is not independent of the order in which the plates are placed.

The Auxiliary Function

Before entering on a discussion of the properties of special systems it is convenient to exhibit an expanded form into which the Δ's may be cast. For this purpose an auxiliary function $D_{h,k}$ is introduced, defined as the determinant obtained by replacing the -1's in Δ by $-s_j$, j being the suffix carried by the r and t of the same column, where

$$s_j = r_j + t_j \qquad \ldots\ldots(18).$$

The equations satisfied by the D's are then

$$\left.\begin{aligned} D_{2m,k} &= D_{2m+1,k}\, r_{2m} - D_{2m+2,k}\, t_{2m}\, t_{2m+1} \\ D_{2m-1,k} &= D_{2m,k}\, r_{2m-1} - D_{2m+1,k}\, s_{2m-1}\, s_{2m} \end{aligned}\right\} \qquad \ldots\ldots(19),$$

corresponding to equations (5) for the Δ's, and

$$\left.\begin{aligned} D_{h,2m} &= D_{h,2m-1}\, r_{2m} - D_{h,2m-2}\, s_{2m-1}\, s_{2m} \\ D_{h,2m+1} &= D_{h,2m}\, r_{2m+1} - D_{h,2m-1}\, t_{2m}\, t_{2m+1} \end{aligned}\right\} \qquad \ldots\ldots(20),$$

corresponding to equations (6). These may be rearranged as

$$\begin{aligned} D_{2m-1,k} + D_{2m,k}\, t_{2m-1} &= s_{2m-1}\,(D_{2m,k} - D_{2m+1,k}\, s_{2m}) \\ &= s_{2m-1}\,(-\,t_{2m})\,(D_{2m+1,k} + D_{2m+2,k}\, t_{2m+1}) \qquad \text{.....(21),} \end{aligned}$$

and $$\begin{aligned} (D_{h,2m+1} - D_{h,2m}\, s_{2m+1}) &= (-\,t_{2m+1})\,(D_{h,2m} + D_{h,2m-1}\, t_{2m}) \\ &= (-\,t_{2m+1})\, s_{2m}\,(D_{h,2m-1} - D_{h,2m-2}\, s_{2m-1}) \qquad \text{.....(22),} \end{aligned}$$

from which it follows that

$$\left.\begin{aligned} D_{2m-1,k} + D_{2m,k}\, t_{2m-1} &= S_{2m-1,k'}\, T_{2m,k''} \\ D_{2m,k} - D_{2m+1,k}\, s_{2m} &= T_{2m,k''}\, S_{2m+1,k'} \end{aligned}\right\} \qquad \text{.....(23),}$$

$$\left.\begin{aligned} D_{h,2m+1} - D_{h,2m}\, s_{2m+1} &= T_{h',2m+1}\, S_{h'',2m} \\ D_{h,2m} + D_{h,2m-1}\, t_{2m} &= S_{h'',2m}\, T_{h',2m-1} \end{aligned}\right\} \qquad \text{.....(24),}$$

where k' and k'' are respectively the odd and even integers next below $k+1$, h' and h'' the odd and even integers next above $h-1$, and

$$S_{i,j} = s_i s_{i+2} s_{i+4} \ldots s_j \qquad \text{.....(25),}$$

i and j being both odd or both even. By eliminating the D's in (23) or (24) with the odd or those with the even suffixes and applying the formula with successive values of m, the expressions

$$\left.\begin{aligned} D_{h,k} &= \sum_i T_{h',i-2} S_{h'',i-1} r_i T_{i+1,k''} S_{i+2,k'} \\ &= \sum_j S_{h'',j-2} T_{h',j-1} r_j S_{j+1,h'} T_{j+2,k''} \end{aligned}\right\} \qquad \text{.....(26),}$$

are obtained, where i denotes all odd, and j all even integers from $h-1$ to $k+1$ inclusive, it being understood that all factors with a suffix outside the limits h to k have the value unity. It is to be observed that when any term is written out in full and the factors are arranged in the order of their suffixes, these factors are alternately $-t$ and s if the factor r is omitted, and that the factor corresponding to the same medium as r is always $-t$ and never s.

Non-absorbing Systems

The value of the expressions for the D's will be realised by taking a simple case. Suppose that all the light incident upon each surface is either regularly reflected or refracted. Then from the definition (18) it follows that $s_j = 1$, and D and Δ are identical. If further the transmissive coefficients for each surface are the same for the two directions in which the light travels,

$$t_{2m} = t_{2m+1},$$

and (26) gives

$$\frac{D_{0,2n}}{-\,T_{0,2n}} = \Sigma \frac{r}{t} \qquad \text{.....(27),}$$

the summation extending over all the surfaces. Also either this formula or equation (23) gives

$$D_{1,2n} = D_{0,2n} - T_{0,2n} \qquad \text{.....(28),}$$

as would be expected. Under the special conditions named, which have been assumed throughout in the earlier investigations already mentioned, we may state these conclusions in the simple law

"In a pile free from absorption the sum of the intensities of the reflected and transmitted beams is equal to the intensity of the incident beam, and the ratio of the intensity of the reflected to that of the transmitted beam is equal to the sum of the corresponding ratios for the component plates or surfaces of the pile."

Such a pile evidently forms an exception to the rule (*d*) that the reflective coefficient depends on the order in which the plates are arranged.

Absorbing Systems

When all the s's are not equal to unity, D and Δ are no longer identical, but it is convenient to express the latter in terms of D's owing to the simple form these functions take under the usual assumptions. It may be remarked that the formulas (5) and (6) are essentially those which give the power of a lens in terms of its component powers and separations, and D corresponds to the special case when the separations are zero. This would be expected since Δ is defined as a continuant, and one of the best known forms for the power is of this character. This observation will prepare us for an expression for Δ resembling Cotes' formula in geometrical optics.

The comparison of equations (5) and (6) with (19) and (20) shows that the laws upon which the D's and Δ's are constructed only differ when transfer is made from one surface to another. For individual surfaces the D's and Δ's are identical. The Δ's for a number of surfaces can be built up according to equations (5) and (6) and the combined quantities expressed in terms of D's simultaneously developed according to equations (19) and (20). The result is

$$\Delta_{h,k} = D_{h,k} + \Sigma\, D_{h,i-1}\,(s_i s_{i+1} - 1)\, D_{i+2,k} + \Sigma\Sigma\, D_{h,i-1}\,(s_i s_{i+1} - 1)\, D_{i+2,j-1}\,(s_j s_{j+1} - 1)\, D_{j+2,k} + \dots \qquad \dots\dots(29),$$

where in the first sum i receives in turn all odd values between h and $k-1$, in the second sum all odd values between h and $k-3$, while j receives all odd values between $i+2$ and $k-1$, and so on. The process is continued until the interval h, k has been divided up into pairs according to the media with corresponding factors of the form $(s_{2m-1}s_{2m}-1)$, the last term consisting of such factors together with $D_{h,h}$ if h is even, and $D_{k,k}$ if k is odd.

We may now apply these formulas to the expressions for the reflective and transmissive factors for the system on the suppositions that losses by absorption only take place within the body of the medium, and that the various factors do not depend on the direction in which the light is travelling. That is to say

$$\tau_{2m} = \tau_{2m+1}, \qquad \rho_{2m} = \rho_{2m+1},$$
$$\rho_h + \tau_h = 1, \qquad \eta_{2m} = \eta_{2m-1},$$

or if even numbers are retained for the surface constants and odd numbers for the medium constants:

$$\left.\begin{aligned} s_0 = s_{2n+1} &= 1, & s_{2m} = s_{2m-1} &= \eta_{2m-1} \\ r_{2m} &= \rho_{2m}\eta_{2m-1}, & t_{2m} &= \tau_{2m}\eta_{2m-1} \\ r_{2m+1} &= \rho_{2m}\eta_{2m+1}, & t_{2m+1} &= \tau_{2m}\eta_{2m+1} \end{aligned}\right\} \qquad \dots\dots(30).$$

On substituting these values in (26), it is seen that the factors η for each side of every surface and $-\tau$ for each surface are common, and the remaining factor in $D_{2h,2k}$ is

$$\sum_{j=h}^{k} \frac{\rho_{2j}}{\tau_{2j}} = \sum_{h}^{k} \frac{r_{2j}}{t_{2j}} = Q_{2h,2k} \qquad \text{.....(31),}$$

and in $D_{2h+1,2k}$ is

$$1 + Q_{2h,2k} = R_{2h,2k}, \text{ say} \qquad \text{.....(32).}$$

T consists solely of the factors $(-t)$, that is to say $(-\tau)$ and η. Let

$$\epsilon = \frac{1}{\eta} \qquad \text{.....(33),}$$

and

$$E = \epsilon_1 \epsilon_2 \dots \epsilon_{2n-1} \qquad \text{.....(34).}$$

Then by (14), (15), and (29), if the intensity of the transmitted light is represented by E, that of the incident light will be represented by

$$\begin{aligned} &R_{0,2n} + \Sigma R_{0,2h} (\epsilon^2_{2h+1} - 1) R_{2h+2,2n} \\ &\quad + \Sigma\Sigma R_{0,2h} (\epsilon^2_{2h+1} - 1) R_{2h+2,2i} (\epsilon^2_{2i+1} - 1) R_{2i+2,2n} \\ &\quad + \Sigma\Sigma\Sigma R_{0,2h} (\epsilon^2_{2h+1} - 1) R_{2h+2,2i} (\epsilon^2_{2i+1} - 1) R_{2i+2,2j} (\epsilon^2_{2j+1} - 1) R_{2j+2,2n} \\ &\quad + \dots \end{aligned} \qquad \text{.....(35),}$$

and that of the reflected light by replacing the R's in equation (35) which have the suffix o by the corresponding Q's. If the light is incident on the other side of the pile the only change is that in the expression for the reflected light the R's with the suffix $2n$ are to be replaced by Q's, that is to say, are to be decreased by unity.

It will be noticed that all the non-vanishing terms in (35) are essentially positive, and that the properties of the components are most conveniently taken in the form of the ratio of the intensities of the reflected to the transmitted light, and the reciprocal of the absorption factor.

DISCUSSION

Mr R. Kingslake: I am very interested in the results which Mr Smith has deduced in connection with the old problem of the pile of plates, but in particular am I intrigued by the formula (27). This appears at first sight to give absurd results, for if r/t for each surface were 1/2, that is, if each surface transmits twice as much light as it reflects, and if there are two plates with four surfaces in all, the formula tells us that the whole pile will finally *reflect* twice as much light as it *transmits*. However, if this particular case is followed out in detail, including all the internal reflections and summing the infinite geometric series so obtained, it will be seen at once that this result is correct in spite of its apparent absurdity.

NOTE ON THE IMMUTABILITY OF TRANSMISSIVE FACTORS WITH REVERSAL OF LIGHT

By D. S. PERFECT, M.A., D.Phil.

(Optics Department, The National Physical Laboratory.)

MS. received, 23rd June, 1926. Read, 24th June, 1926.

ABSTRACT. Direct experimental evidence has been obtained that the transmissive factor of the surface separating two media is unaltered if the direction in which the light travels is reversed.

In discussing the transmission of light by a system of media, it is customary to assume that the coefficients of transmission, absorption, and scattering are equal in both directions. The justification for these assumptions hitherto has probably not been very direct. From an analysis* (undertaken as part of a larger investigation on the properties of partially reflecting systems) in which these assumptions were not made it appears that these properties may be directly inferred from the equality of the transmissive coefficients of unsymmetrical systems for the direct and the corresponding reversed path. Measurements of this kind at an incident angle of 45° have been made by visual photometry on a series of silver films deposited on glass. The transitions at the surfaces are thus, in the one direction, air to glass, glass to silver, silver to air, and in the other air to silver, silver to glass, glass to air. In all cases, over a wide range of densities of silver (in which the transmissive coefficient varied from 0·18 to 0·80), it was found that the transmission coefficients were equal within the accuracy of the measurement, thus verifying the assumptions by direct experiment.

* T. Smith, "On the Light Transmitted and Reflected by a Pile of Plates," *Trans. Opt. Soc.* 27 (1925–26) 317.

REVIEWS

Photometry. By J. W. T. Walsh, M.A., M.Sc., A.M.I.E.E., F.Inst.P. Pp. xxvii + 505; 303 Figs. (London: Constable and Co., Ltd., 1926.) Price 40*s*. net.

Mr Walsh's book is divided into the following chapters: Historical Note, Radiation, The Eye and Vision, The Principles of Photometry, Standards of Candle-Power, Measurement of Candle-Power in a single direction, Candle-Power Distribution and Total Flux Measurement, Heterochromatic Photometry, Spectrophotometry, The Measurement of Colour, Physical Photometry, Illumination Photometry, Measurement of Brightness and of Reflection and Transmission Factors, Photometry of Projection Apparatus, Stellar Photometry, The Photometric Laboratory. Each chapter is furnished with a bibliography, the total number of entries being several thousand; there are several appendixes and a full and orderly index. The production of the book is admirable.

This elaborate treatise is a store-house of facts, and yet it is not a mere compilation. Mr Walsh does not aim simply at bringing within a single cover everything valuable that has been written on his subject; he selects what lies within his experience and what he judges to be good. In all that concerns the experimental methods of standard visual photometry this is probably a distinction without a difference; what Mr Walsh does not know about this subject is not knowledge. Even if it were suggested that rather more information might be given about methods practised elsewhere than at the National Physical Laboratory, Mr Walsh might reply that, whether other methods are better than his own or worse, his are those to which everyone in this country has to conform.

In short, criticism of this part of the work from anyone whose experience is not as extensive as that of the author would be impertinent. The only suggestion that we can make is that Mr Walsh might have extended the circle of his potential readers if he had set his standard rather lower. There are probably not 20 persons in the country who ever aspire to make photometric measurements of the highest accuracy; there are a very much larger number, for instance in industrial test-rooms, who aim at a much lower accuracy and yet are more in need of information and instruction. If they are not frightened away altogether by a treatise of this magnitude, they will probably turn eagerly to the last chapter. But we fear they will be disappointed. There are two forms of the experimental art; one consists in designing the very best apparatus; the other in making the best of a bad one. Mr Walsh's interests do not extend to the latter form. The failings of commercial electrical instruments, devices for finding and eliminating bad contacts, the gain of speed at the expense of accuracy—on all such matters, vital to the largest class of photometricians, he preserves a lordly silence.

Since the length of even the most ponderous volume is limited, it is well to accompany suggestions for additions by suggestions for omissions. The value of the main part of Mr Walsh's book arises from his intimate and detailed knowledge of his subject; he is unwise to dilute it by venturing further than necessary beyond that knowledge. Some subsidiary matters cannot be avoided, such as colour; we cannot judge whether the treatment of this very difficult and controversial subject will commend itself to those who have studied it closely. On the other hand, there was no need to say anything about stellar photometry; though we do not profess to know much more about this subject than Mr Walsh, we can hardly believe that the seven pages he devotes to it will be of any service to astronomers. Nor was there any need to touch on "physical methods"; for, by Mr Walsh's own confession, they are at present confined wholly to problems other than those to which visual photometry is applied. What is the use in this chapter of five pages on bolometers

and radiomicrometers and of two on photographic photometry? Not even the bibliography is complete; for each of these topics has a literature comparable with that of all visual photometry. Moreover in the parts of this chapter where we can claim special knowledge, the information is so superficial as to be misleading, and one important paragraph (page 226, lines 11–27) is definitely false. Again why waste the thirty-one pages of the chapter on Radiation in an exposition of elementary matters which are better (if only because more fully) treated in any university text-book on optics?

To this last question Mr Walsh gives an answer. It is "that the practice of a branch of applied science cannot be properly carried on without some grasp of the theory underlying it." We agree most heartily that photometry is a science as well as an art, that it has an intellectual as well as a practical interest, and that it is as important for the practical as well as for the intellectual interest that its basis should be understood. But we differ altogether from Mr Walsh in what constitutes that basis. Theory is not the basis; every theory discussed by Mr Walsh in his chapter on Radiation might be abandoned without any consequent effect on photometric methods. Nor is it mere convention, as Mr Walsh seems to suggest when he bases his discussion of the Principles of Photometry on the definitions formulated by an International Committee. The basis of photometry, and of every system of measurement, is laws, which neither theories nor committees can alter; and our chief quarrel with Mr Walsh's book is that it contains no adequate statement of the laws which underlie photometry in the sense that, if they were not true, photometry would be impossible.

Of these laws there are two of primary importance. The first is that illumination is additive. Of this law we can only find one statement (page 253); it is merely incidental and might easily be overlooked; and it is deprived of any value it might otherwise possess by the utterly false implication that all magnitudes are additive. Even in the chapter on Illumination Photometry, the law is never more than hinted. The second is the law of the inverse square. This is stated where it might be expected (page 95), but with only one of the necessary conditions and that the one which is least worth stating explicitly. For since points are the only geometrical figures which possess distances, the mention of distances confines the application of the law to points. The second, and as Mr Walsh himself insists, the more important condition, that there shall be no reflecting surfaces in the neighbourhood, does not appear until page 170, after the statement of the Principles is complete. The third condition, that the medium shall be optically homogeneous and non-absorptive, is never mentioned at all, even in the chapter on Transmission. And the only proof of the law which is offered is a single reference in the bibliography to two papers, fifty years old and difficult of access, of which not even a summary is given in the text. It is thus that Mr Walsh instructs his readers in the things which "underlie" photometry.

In this matter Mr Walsh is no worse than many of his predecessors; Lambert, the first photometrician, is perhaps the only one who is notably better. But he has less excuse than most of them; for he has deliberately professed an intellectual interest in his art. It is for that reason that we have attacked him so strongly on a matter that will seem trivial to many. But we suggest that it is not so trivial as it is usually thought; a knowledge of principles is unnecessary while an art is static; it becomes essential when advances are to be made. We fear that Mr Walsh's attitude towards photometry will hinder rather than help the great developments which he promises us in the near future.

A reviewer is almost always forced to devote more of his space to blame than to praise. But we would not end upon that note. Mr Walsh has written a very valuable book on the existing methods of standard visual photometry; and all who are concerned with that work are in his debt.

NORMAN R. CAMPBELL.
BERNARD P. DUDDING.

Experimental Optics. A Manual for the Laboratory. By G. F. C. SEARLE, Sc.D., F.R.S. Pp. xvi + 357; 236 Figs. (Cambridge: University Press, 1925.) Price 16*s.* net.

This manual, based on laboratory work in experimental optics carried out in Dr Searle's practical class at the Cavendish Laboratory, will be welcomed by teachers and students of physics.

Starting with a concise statement of Maxwell's electromagnetic theory of light, the author describes an extensive set of experiments dealing with the reflection and refraction of spherical waves at plane and spherical surfaces. A chapter of thirty-five pages is devoted to astigmatism due to mirrors and lenses, curvature of the field, and the effect of stops. The last two chapters contain a concise account of the theory of interference, diffraction, and polarisation by reflection, with experimental illustrations.

Of the ingenuity and skill in the development of the experimental work we cannot speak too highly. The apparatus described is in general of a very simple character, and is capable of being made in an ordinary laboratory workshop. Examples of practical work carried out by Dr Searle and his pupils are appended to every experiment, indicating how the results should be written out and forming a guide to the degree of accuracy of measurement which should be obtained.

Out of seventy experiments, the author has allotted fifty-six to lenses, mirrors, and prisms. Of the remaining fourteen, one is concerned with the determination of the polarising angle, while the others deal with simple interference and diffraction phenomena. No mention is made of refractometers with their various applications, or of the setting up of interferometers and their use in the testing of lens surfaces. Quantitative photometric measurements are of fundamental importance in many branches of modern physics, and in view of recent developments in technique and apparatus for such measurements it seems surprising that no reference is made to this important branch of optics.

In spite of the pressing need of restricting the size and price of students' books, we venture to suggest that in a future edition space may be found for more experiments in interferometry and photometry and for more detailed work in spectroscopy. A few of the numerous examples on interesting but somewhat unpractical points connected with lenses might well be omitted. We wonder how often the practical worker on optics makes use of the properties of the Bavais points.

The measurement of wave-lengths by means of Newton's rings is an interesting illustration of theory, but is hardly suitable for the accurate determination of wave-lengths. It would be more in accordance with the trend of modern teaching to reverse the procedure, viz., to assume a knowledge of the wave-length of the incident light and to deduce a value for the curvature of the spherical surface.

The chapter on polarisation might with advantage be extended to include an account of refractive indices and the properties of crystals. Physicists use refracting material other than glass; crystals are not nowadays the monopoly of mineralogists, and the measurements of their simpler properties not only elucidate theory but provide excellent and interesting tests of manipulative skill.

In spite of these omissions, Dr Searle has given us an original and practical textbook on experimental optics which can safely be recommended to the physicist and to the optician.

I. O. GRIFFITH.

ABSTRACTS OF PATENT SPECIFICATIONS

241690. Weigel, A. PROJECTION APPARATUS.

The apparatus is designed for use before or behind the screen. The axis of the projector is vertical, and the objective and the source of light are interchangeable. An inclined mirror is fitted which can be set to throw the light in either of two opposite lateral directions, so as to obtain lateral reversal of the image when necessary. The condenser and slide are cooled by circulating over them a current of cooled and filtered air.

242468. Wilson, J. S. and another. TACHEOMETRIC TELESCOPES.

In an instrument of the type focussed by moving an internal negative lens, this lens is of the same (but opposite) focal power as the objective. The axis of movement is halfway between the objective and the system of marks, the arrangement being such that the angles subtended by the object at the principal point of the objective and by the stadia intercepts at the focussing lens closely approximate to the angle subtended by the object at the anallatic point.

242576. Tillyer, E. D. MARKING LENSES.

Phosphoric acid applied to a polished lens surface and allowed to stand produces a permanent marking, which is normally invisible, but can be seen after breathing upon the glass.

242626. Zeiss, C. SIGHT-TESTING APPARATUS.

Apparatus for the objective measurement of refraction includes an observation device fitted with a diaphragm, the image of which is formed in the pupil side by side with the image of the light aperture. The settings required to obtain a standard appearance enable a measure to be obtained for symmetrical and astigmatic effects.

242627*. Jarnier, J. H. STEREOSCOPY.

In stereoscopic projection, paired black images are thrown upon the screen successively by light of complementary colours, observation being made through suitable coloured spectacles.

242631*. Goerz Optische Anstalt Akt.-Ges. RANGEFINDERS.

Rangefinders using the height above sea level, or the height of the target as base, include a telescope the elevation of which is controlled by an exponential cam, giving direct readings upon a logarithmic scale. A correction cam may be set to allow for the curvature of the earth and for atmospheric refraction.

242686. Dorey, W. A. VEHICLE LAMPS.

The inner member of the lamp front is formed with concentric prisms to reduce the divergence of the rays, and the front member carries straight prisms to concentrate the light in one plane, while permitting divergence in a plane at right angles.

242826. Watts, E. R. and Sons, Ltd., and another. FINE ADJUSTMENTS.

In a differential screw device adapted for setting a level, one member of the screw carries a part which engages with a head fixed to the level tube, so that when these parts are pressed together by a spring, no relative rotation can ensue.

* Specification published before acceptance.

243279. Herbert, J. F. OPHTHALMIC APPARATUS.

A cup-shaped member adapted to fit against the eye, and to be held in place by the eye-lids, carries in an extension piece light bulbs to illuminate the eye, and a lens to enable it to be examined or photographed.

243893. Liesegang, F. P. PROJECTION APPARATUS.

In an episcopic apparatus the object is illuminated by one or more lamps fitted with a condenser and backed by a reflector, and the upper parts of the casing are polished or fitted with mirrors to throw back upon the object light which is scattered from its surface.

244380. Tallack, H. T. LEVELS.

In addition to the spirit and the air-bell, the tube contains a quantity of mercury so proportioned that a minimum effect is produced by temperature changes upon the size of the bubble.

244407*. Quartzlampen Ges. TESTING BY ULTRA-VIOLET LIGHT.

The main compartment of a casing encloses a source emitting ultra-violet light, which passes through a filter on to the object to be examined, which is placed upon an observation table. A sight aperture enables the object to be viewed directly, and the light transmitted may be observed in a mirror, or allowed to act upon a photographic plate.

244448*. Zeiss, C. PROJECTION APPARATUS.

This specification relates to a device for projecting upon a hemispherical screen the fixed-star sky, together with images of the sun, moon, and planets, and lines such as the ecliptic and the meridian, all the projectors acting as if the light came from a single point. Arrangements are made by which the axis of rotation of the main system may be adjusted for the latitude of the place of observation.

244495*. Goerz Optische Anstalt Akt.-Ges. GUN-SIGHTS.

A telescopic gun-sight is adapted for use by direct sighting or for setting by level, and has in addition an indicator for setting the range. The elevating gear is operated by a spindle which is surrounded by a sleeve capable of being clutched or declutched at will, according to the system of sighting being followed.

244514. Cunynghame, Sir H. H. CAMERA LUCIDAS.

A picture to be copied is held vertically in a frame which is hinged to a horizontal frame carrying a sheet of paper. A plate of glass placed obliquely between the two frames produces a reflected image which can be drawn upon the paper.

244748*. Wolf Ges., G. OPTICAL INSTRUMENTS.

A gastroscope consists of a bent tube which is fitted at one end with a lamp projecting light through two lateral windows, and an optical system viewing through a window near the elbow. This system is used only when inserting the instrument, and is then replaced by a system viewing laterally through a window in the straight part of the tube.

244849. Long, E. M. GRINDING AND POLISHING LENSES.

The lens is treated by a rotating lap, across which it is oscillated. The lap is rotated from beneath by a suitable clutch and gearing, the lens being moved across it by a rack and pinion device, operated by an eccentric of variable throw.

* Specification published before acceptance.

245108*. Marsat, A. PROJECTION APPARATUS.

A transparent body with parallel faces, or of prismatic form, is placed between the spectators and the screen, so as to produce a relief effect upon the image. A thick plate of glass, an assemblage of thin plates, or a vessel filled with water or gas may be employed.

245336. Weisz, H. LIGHT FILTERS.

In order to reduce the influence of grinding inaccuracies, the amount of colouring matter in the melt is such that the correct depth is obtained on grinding down to about 2 mm. thickness. Two or more such plates, with or without a gelatine filter between them, may form a composite filter.

246192. Halsbury (H. Goulburn), Earl of. TESTING LAMPS.

A screen furnished with a horizontal slit is placed successively at different positions in front of the glass of a projection lamp, and the deviation of that portion of the light from the horizontal, or from the main beam is measured. The form of the prismatic element which must be applied to the lamp front at that point in order to give the required distribution of light can then be determined. The method can also be adapted to ascertain the appropriate curvature at each point of the projection mirror.

246270. Carrol, W. C. PROJECTION APPARATUS.

In an apparatus for daylight projection, a screen of translucent absorbent material is kept saturated with an oily substance by a range of wicks which feed the oil to a felt border surrounding the screen. A continuous film passes through the projector, which is enclosed in the casing upon which the screen is mounted.

246575. Trezise, W. E. LENSES.

In order to produce an image giving a stereoscopic effect, there is employed in conjunction with the lens a diaphragm formed with two or more spaced apertures, and behind the lens a series of similar diaphragms is placed each having apertures corresponding to those in the first.

247184. Soc. Anon. Française Holophane. VEHICLE LAMPS.

The inner face of the glass is formed with a central series of horizontal prisms, and one or more outer annular series of progressively smaller angle. The outer face is provided with similar series of vertical prisms, arranged as central and outer bands.

247231*. Quarzlampen Ges. ULTRA-VIOLET APPARATUS.

A lamp for treating liquids or gases with ultra-violet light is so arranged that it closes an aperture in the vessel containing the fluid. The lamp is fitted with a reflector consisting of strips of coated porcelain or quartz, preferably of rhomboidal section, and spaced apart to assist cooling.

247606*. Elek, S. TACHEOMETERS.

The telescope of a surveying-instrument is adapted to measure distances directly by fitting in front of the eyepiece two prism-lenses or mirrors to divide the field into portions having optical axes at a given difference of level. A reading is obtained of the apparent position of the vernier in one portion of the field, and compared with the zero reading of the other half.

247884. Kronenberger, B. SIGHT-TESTING APPARATUS.

A hand instrument adapted for self-diagnosis comprises a frame furnished on each side with reflecting or magnifying elements, and provided with movable pointers to

* Specification published before acceptance.

enable the observer to indicate upon the annular rim changes noticeable in the eye. In a modified form, the magnifying and reflecting elements may be mounted side by side, at a separation equal to the inter-ocular distance.

248524. Cooke, L. H., and others. THEODOLITES.

A centering device, which is placed between the angle-measuring portion and the levelling screws of a theodolite, consists of two members, one of which can be freely moved in a horizontal plane, and the other is capable of receiving a direct or slow motion in guides formed on the first member. A thorn on the azimuth centre can be moved up almost into contact with the plumb-bob, and observed through a lens in silhouette against a translucent screen.

248960. Beck, C. J., and others. SPECTROSCOPES.

An image of the slit is projected upon the spectrum to form an index, the position of which may be varied by rotating the prism. The displacement is indicated upon a graduated drum, which thus provides an approximate scale of wave-lengths.

249094*. Optical Development Corporation. GRINDING LENSES.

Simultaneous grinding of a number of lenses is effected in a machine which includes a rank of grinding-wheels beneath which a series of curved members, upon which the lenses are mounted, are carried in succession by a slowly rotating drum. When the operation is completed, the members fall from the drum and pass down guide rails from the machine.

249110. Zeiss, C. EYE-TESTING APPARATUS.

An instrument for testing the field of view of the eye of a subject includes two optical systems for projecting images of a mark on to a curved surface, one system being fixed, the other being adjustable to cause the image to travel round the surface. A reading is taken of the position of the second system when, while observing the fixed mark, the other just disappears.

249122*. Metallique Spolecnost, etc. REFLECTORS.

In the production of specula, mirrors, and like reflectors, a layer of water-glass and caustic potash lye is applied between the reflecting layer and the protecting metallic coating.

249580. Linn, J. D. SCALE-READING DEVICES.

To facilitate the reading of sub-divisions of the scale of an instrument, an image of the graduation is projected upon a translucent plate marked with a diagonal scale, and capable of receiving a measured displacement by rotating an eccentric ring.

250268*. Kitajima, I. LENSES.

A variable-power lens system is obtained by superimposing crosswise, and sliding upon each other, two or more cylindrical lenses, each of which has one surface of constant curvature, the other surface being of conical form, increasing in curvature from one end to the other.

250737. Eden, J. L. RANGEFINDERS.

Two arms, one of which carries fixed and movable prisms, are pivoted together and adjusted by a micrometer screw in such a way as to form a triangle similar to that formed by the objective and the two prisms. The range is read off directly from the micrometer.

* Specification published before acceptance.

251018. Crow, A. B., and another. OPTICAL SYSTEMS.

A number of achromatic prisms, with their thin edges meeting, are arranged in conjunction with an objective to produce identical images around the principal axis. Partitions are fitted to prevent overlapping of the images, and provision may be made for varying the effective area of the prisms.

251027. Astafiev, M. OPTICAL SYSTEMS.

A number of triangular prisms, capable of being assembled to form a cube, are furnished with reflecting strips or areas in such a way that multiple images parallel to three faces of the cube are formed, one by transmission, the others after a single reflection only.

251281*. Oehmichen, E. PROJECTION APPARATUS.

A lamp chamber and condenser are adapted to fit on to the lens portion of a camera to enable pictures and negatives to be projected on to a screen. In front of the objective, a removable fitting including an auxiliary lens and a screen may be placed to enable an image to be produced and examined at close quarters.

251654. British Thomson-Houston Co., Ltd. PHOTOMETRIC APPARATUS.

The apparatus includes two target surfaces, the light reflected from which is compared by a photometer of the type which includes an objective, polariser, analyser, and eyepiece. One of the target surfaces is arranged upon an illuminating surface, the other upon the surface the reflecting power of which is under test.

252159*. Goerz. Opt. Anstalt Akt.-Ges. RANGEFINDERS.

The movement of the sleeve setting the ranging prisms is transmitted to an indicating disc through logarithmic cams, and a second disc, moving over the first, is similarly set in accordance with the logarithmic sine of the angle of elevation. An additional system may also be fitted to indicate the horizontal projection of the position of the target.

252387. Metropolitan Vickers Electrical Co. TELEVISION APPARATUS.

An image of the object is focussed upon a mirror enclosed in a vessel filled with liquid of the same refractive index as the glass backing of the mirror, and the light is reflected to and fro between this mirror and a moving mirror which is oscillated by means of a pair of piezo-electric crystals controlled by an alternating current generator. The light then passes to a third mirror which is oscillated at a much slower speed by an electromagnet, and then on to a photo-electric cell in the transmitting circuit. At the receiving end a synchronised set of mirrors is employed to reproduce the image.

252639. Martin, K. OPTICAL SYSTEMS.

Dual coloured images arranged side by side and side uppermost are projected by a pair of objectives and two systems each consisting of three right-angled prisms cemented together. By this means the two images are separated and turned through a right angle.

252696*. British Thomson Houston Co., Ltd. TELEVISION APPARATUS.

A transmission current of varying intensity is produced by exploring a projected image by means of a cathode ray, which is electrically deflected to describe a spiral or zig-zag path. At the receiving end, a cathode ray is generated, the intensity of which is varied with the transmission current, and which is moved in synchronism with that at the transmitting end.

R. J. T.

* Specification published before acceptance.

PROCEEDINGS

AT THE

MEETINGS OF THE OPTICAL SOCIETY

1925–26

15th October, 1925.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Elections: (Membership) Dr E. H. Rayner.
(Student Membership) Mr J. H. Lambert-Smith.

The following papers were read:

"On Brewster's Bands." Part I. By Professor C. V. RAMAN, F.R.S., and SUSHIL KRISHNA DATTA, M.Sc. (Read by Mr F. F. S. BRYSON.)

"Lagrange's Theorem and Stationary Functions." By T. SMITH, M.A., F.Inst.P.

Mr W. G. COLLINS demonstrated some new instruments for recording rapidly varying phenomena.

12th November, 1925.

Extraordinary General Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

After the President had briefly explained the reasons why the Council considered it necessary to reframe and alter the existing Rules, a resolution was moved from the Chair that, due notice in accordance with Rule XXVI having been given to all members, the old Rules be replaced by the new set of Rules as printed and circulated. The motion was carried *nemine dissentiente*.

12th November, 1925.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Elections: (Membership) Mr H. Holloway, Mr C. H. Keeler, Mr J. Mansell, and Mr R. C. Nurnberg.

The President briefly referred to the proposal to form an Ophthalmological Section and to the importance of ophthalmological research, after which he invited Mr H. H. Emsley to take the Chair.

The following papers were read:

"Some Recent Improvements in Modern Ophthalmic Lenses." By S. A. EMERSON, A.R.C.S., D.I.C., B.Sc.

"The Relation of Visual Acuity and Accommodation to Ametropia." By W. SWAINE, B.Sc. (Read by Mr H. H. EMSLEY.)

"Irregular Astigmatism of the Eye: Effect of Correcting Lenses." By H. H. EMSLEY, B.Sc.

"Some Causes of Apparent Astigmatism of the Eye other than Cylindrical Errors of Refraction." By E. F. FINCHAM.

10th December, 1925.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Elections: (Membership) Mr H. Buckley, Prof. A. O. Rankine, Prof. R. Schmehlik, Mr J. W. T. Walsh, and Dr A. F. A. Young.
(Corporate Membership) Messrs Chance Brothers and Co., Ltd.

The following papers were read:

"General Principles of Photographic Surveying." By Colonel H. S. L. WINTERBOTHAM, C.M.G., D.S.O., R.E.

"Perspective Conditions of Photogrammetry." By Lt.-Colonel M. N. MACLEOD, D.S.O., M.C., R.E.

"Some Precision Problems in Air Survey." By Lieutenant M. HOTINE, R.E.

5th, 6th and 7th January, 1926.

Sixteenth Annual Exhibition of Electrical, Optical and other Physical Apparatus held at the Imperial College of Science, jointly with the Physical Society of London.

The following discourses were given:

"The Search for Ultra-microscopic Organisms." By J. E. BARNARD, F.R.S.

"The Mechanical Design of Instruments." By Professor A. F. C. POLLARD, A.R.C.S., A.M.I.E.E.

"Electrical Listening." By Major W. S. TUCKER, D.Sc.

21st January, 1926.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Elections: (Membership) Lt.-Col. A. D. Carden, Capt. D. S. C. Evans, and Mr A. W. Smith.
(Student Membership) Mr J. E. Elkan, Mr H. Spurling, and Mr S. Weinfeld.

Messrs R. Kingslake and T. M. Rose were elected Hon. Auditors for the Society's accounts for the year 1925.

The following papers were read:

"The Interferometer Patterns due to the Primary Aberrations." By R. KINGSLAKE, A.R.C.S., B.Sc.

"A Trichromatic Colorimeter suitable for Standardisation Work." By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S.

"On a New Method of Colorimetry." By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S.

"A Criticism of the Monochromatic-plus-white Method of Colorimetry." By J. GUILD, A.R.C.Sc., F.Inst.P., F.R.A.S.

Messrs THE GAUMONT CO., LTD., gave a demonstration of an optical projection device for lecture purposes, for use in connection with an improved type of daylight screen.

11th February, 1926.

Annual General Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

The Hon. Secretary presented the Annual Report* of the Council, which was approved.

* See p. ix.

The Treasurer's Report* and Balance Sheet† were presented by the Hon. Treasurer, and were approved.

The Chairman read the list of persons nominated by the Council for office in the year 1926–27 and declared all these duly elected as no other nominations had been received:

President: Mr T. SMITH.
Vice-Presidents: Instr.-Comdr. T. Y. BAKER, Mr F. F. S. BRYSON, Dr R. S. CLAY, Mr H. H. EMSLEY.
Business Secretary: Prof. A. F. C. POLLARD.
Papers Secretary: Mr F. F. S. BRYSON.
Treasurer: Major E. O. HENRICI.
Editor: Dr JOHN S. ANDERSON.

Council:

Already serving:		Elected, February, 1926:
Mr D. BAXANDALL.	Mr J. GUILD.	Mr E. T. HANSON.
Mr W. M. BRETT.	Mr H. C. RAXWORTHY.	Dr G. F. C. SEARLE.
Mr W. B. COUTTS.	Mr J. RHEINBERG.	Mr W. SWAINE.
Mr A. H. EMERSON.	Mr F. C. WATTS.	
Mr E. F. FINCHAM.		

Votes of thanks to the retiring Officers and Members of Council, to the Hon. Auditors, and to the Governors of the Imperial College, with Prof. Callendar, for the use of the College premises for the meetings of the Society were unanimously passed.

The President for the time being of the Optical Society of America was elected an Ex-officio Fellow.

In accordance with Rule 86, Major E. O. Henrici and Mr T. Smith were appointed Trustees of the Society's funds.

11th February, 1926.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Elections: (Membership) Mr W. B. Barker, Prof. L. H. Cooke, Mr F. Lancaster Jones, and Mr S. C. Laws.

(Fellowship) Mr A. W. Angus, Mr D. Baxandall, Mr M. Blood, Mr W. M. Brett, Mr J. H. Dowell, Mr E. F. Fincham, Mr E. T. Hanson, Instr.-Comdr. N. M. S. Langlands, Mr C. C. Paterson, Mr J. Pike, Prof. A. O. Rankine, Mr H. C. Raxworthy, Mr J. J. Steward, Mr E. W. Taylor, and Mr S. J. Underhill.

The following papers were read:

"The Nature of the Polishing Operation." By F. W. PRESTON. (Read by Mr F. F. S. BRYSON.)

"The Annealing and Re-annealing of Glass." Parts III and IV. By W. M. HAMPTON, B.Sc., A.I.C.

25th February, 1926.

Special Meeting held at the Imperial College of Science.

Mr H. H. EMSLEY, B.Sc., Vice-President, in the Chair.

The Chairman announced that the meeting had been arranged by the Sub-committee

* See p. xii. † See p. xiv.

appointed by the Council to consider and report on the proposal to form an Ophthalmological Section of the Society, in order to assist them to arrive at an estimate of the interest of the members of the Society in ophthalmological optics.

The following papers were read:

"Notes on the Significance and Detection of Low Errors of Refraction." By O. AVES.
"Vertex Power and its Measurement." By W. H. A. FINCHAM.

11th March, 1926.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Elections: (Membership) Mr F. E. Altman, Mr S. A. Emerson, and Mr G. A. Graveney.
(Fellowship) Mr W. B. Barker, Dr H. E. Ives, Mr J. W. Perry, Dr E. H. Rayner, and Mr T. M. Rose.

The following papers were read:

"Notes on Some Physical and Optical Characteristics of the Stylus-on-Celluloid Method of Recording." By W. G. COLLINS.
"Application of the Hartmann Test to the Measurement of Oblique Aberrations." By R. KINGSLAKE, A.R.C.S., B.Sc.
"Note on Chalmers' Applications of the Hartmann Test." By T. SMITH, M.A., F.Inst.P.

10th June, 1926.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Election: (Membership) Mr H. W. Lee.

The following papers were read:

"The Distribution of Light in Elementary Optical Images." By Professor L. C. MARTIN, A.R.C.S., D.I.C., D.Sc.
"Note on the Criterion for the Best Position of Focus." By T. SMITH, M.A., F.Inst.P.
"The Stationary Value of Axially Symmetric Functions." By T. SMITH, M.A., F.Inst.P.

24th June, 1926.

Meeting held at the Imperial College of Science.

Mr T. SMITH, M.A., President, in the Chair.

Election: (Membership) Mr D. Davis, Lt. M. Hotine, Prof. Felix Jentzsch, Col. Sir Gerald Lenox-Conyngham, Mr B. Long, and Mr C. F. Smith.

The following papers were read:

"Fraunhofer's Work and its Present-day Significance." By Professor MORITZ VON ROHR, Ph.D., Hon.M.D. (Read by Dr JOHN S. ANDERSON.)
"On the Light Transmitted and Reflected by a Pile of Plates." By T. SMITH, M.A., F.Inst.P.
"Note on the Immutability of Transmissive Factors with Reversal of Light." By D. S. PERFECT, M.A., D.Phil. (Read by Mr T. SMITH.)
"The Treatment of Reflection as a Special Case of Refraction." By T. SMITH, M.A., F.Inst.P.

THE OPTICAL SOCIETY

REPORT OF THE COUNCIL TO THE ANNUAL MEETING OF MEMBERS, 11TH FEBRUARY, 1926

THE COUNCIL of the Optical Society presents the following Report for the year ending 31st December, 1925:

During the year nine Ordinary Meetings of the Society were held at the Imperial College of Science and Technology by permission of the Governors and Professor Callendar, to whom the thanks and appreciation of the Society are due. In addition to the Ordinary Meetings, two Extraordinary General Meetings were held, on 11th June and 12th November. At the meeting on 11th June certain reductions of the subscriptions and the suspension of the entrance fee were approved. At the meeting on 12th November revised Rules were adopted.

At the beginning of the year a paper entitled "A Peep into Herschel's Workshop" was read by Dr W. H. Steavenson, and printed as No. 4 of Volume 26 of the *Transactions*. This part of the *Transactions* also contains a valuable catalogue of instruments made and (or) used by Sir William Herschel, as preserved at Slough, which was drawn up and presented to the Society by Dr Steavenson.

On 12th February the retiring President, Emeritus Professor Archibald Barr, delivered the Presidential Address.

At the last meeting of the year a series of important papers on the subject of Photogrammetric Surveying was read and discussed.

Early in the year a Questionnaire on Colour Terminology compiled by the Optical Society of America was received from the President, Dr Herbert E. Ives. The Council appointed a Committee to obtain and analyse replies, but they were unable to make arrangements for the returns to be completed by the time at first contemplated, and the instructions to the Committee have been modified with the object of ascertaining representative British views in another way.

In recent years the activities of the Society have expanded in many directions in the field of optics, and it was felt by the Council that the prevailing interest of many of the members in ophthalmological optics was to a certain extent becoming overshadowed. In order, therefore, that the original object of the Society should be strengthened rather than weakened, the Council appointed a sub-committee to consider the desirability of forming an Ophthalmological Section of the Society. A programme of papers of an ophthalmological character, arranged by this sub-committee, was presented at a well-attended meeting on 12th November, at which Mr H. H. Emsley took the Chair. Another meeting of a similar character will be held at an early date. The Council hope that these meetings will provide sufficient evidence of the interest of members in ophthalmological matters to justify the formation of an Ophthalmological Section.

In conjunction with the Physical Society of London, the Fifteenth Annual Exhibition of Electrical, Optical and other Physical Apparatus was held at the Imperial College of Science and Technology on 7th and 8th January and was largely attended, as in previous years. During the Exhibition discourses were delivered by Mr F. Twyman on "Some Experiments with Interferometers" and Mr C. F. Elwell on "Talking Motion Pictures."

Throughout the year Fellows and Members were invited by the Board of the Institute of Physics to further lectures on "Physics in Industry" and the thanks of the Society are due to the Board for their courtesy.

The Society of Glass Technology extended an invitation to attend a general symposium on the Constitution of Glass, which took place in London on May 20th, and was very much appreciated by members of the Optical Society.

The thanks of the Society are due to the Illuminating Engineering Society who, as in previous years, have issued invitations to all their meetings.

The President and Council of the Royal Microscopical Society organised a Conference at Sheffield on 20th April and the following days at the invitation of the University of Sheffield, the Civic Authorities, the Master Cutler, and the Research Associations. Members of the Optical Society much appreciated the invitation to attend the Conference, at which Mr F. F. S. Bryson acted as the Society's representative.

Mr R. S. Whipple has continued to represent the Society on the National Committee for Physics, and Professor F. J. Cheshire has represented the Society on the Board of the Institute of Physics.

In the last Annual Report reference was made to the surplus funds of the Society and to a special sub-committee appointed by the Council to make further recommendations. As a result the initial work necessary to organise an Optical Convention to be held in 1926 was successful, and the Council were able materially to assist the movement by contributing the sum of £200 to the Guarantee Fund of the Convention, and by bearing certain preliminary expenses necessarily incurred before the Executive Committee of the Convention was formed.

The Report of the Treasurer shows that the financial position of the Society is satisfactory. The Council hope that, in consequence of the reduction in the rates of subscription, the membership of the Society will increase and that thereby a sufficient income may be obtained to permit of the continued extension of the Society's activities.

LIBRARY

The following publications have been received during 1925:

Helmholtz's Treatise on Physiological Optics. Vol. 2, *The Sensations of Vision.* Translated from the Third German Edition. Edited by Prof. JAMES P. C. SOUTHALL.

Die Entwicklungsjahre der Kunst, optisches Glas zu schmelzen. By M. v. ROHR.

A Text-Book of Glass Technology. By F. W. HODKIN, B.Sc., A.I.C., and A. COUSEN, M.Sc., A.R.C.Sc., A.I.C.

A Radiometric Investigation of the Germicidal Action of Ultra-violet Radiation. By W. W. COBLENTZ and H. R. FULTON.

Die Geschichte der Emil Busch A.-G. Optische Industrie, Rathenow, vom Wirtschaftlichen Standpunkt. By Dr KARL ALBRECHT.

French-English and English-French Dictionary of Technical Terms, etc. By J. O. KETTRIDGE, F.S.A.A., A.C.I.S.

Fremdsprachliches Optisches Wörterbuch (deutsch-englisch-französisch-spanisch). Teil I: Das Auge. Herausgegeben von der Deutschen Optischen Wochenschrift. Bearbeitet von Dr W. PIETZSCH.

Critical Potentials associated with Excitation of Alkali Spark Spectra. By F. L. MOHLER.

Illuminating Engineering Nomenclature and Photometric Standards. American Standard, approved April 29, 1925, by American Engineering Standards Committee.

Оптика Бинокля (*Optics of the Binocular*). By B. E. MOURASHKINSKY.

Glass Research Association Bulletin, No. 14 (*Aug.* 1925). *A Review of the Work of the Association*, 1920–1925.

Experimental Optics. By G. F. C. SEARLE, Sc.D., F.R.S.

Practical Photo-Micrography, 2nd edn. By J. E. BARNARD, F.R.S., F.Inst.P., F.R.M.S., and FRANK V. WELCH, F.R.M.S.

Tables Annuelles Internationales de Constantes et données numériques, Vol. 5, 1st Part. Years 1917–1922.

Annual Reports of the Smithsonian Institution for 1922 *and* 1923.

Tokyo Imperial University Calendar, 1924–1925.

British Optical Association Catalogue of Joint Libraries of the British Optical Association and the Optical Society.

The following eight addresses delivered on the occasion of the centenary celebrations of the Founding of the Franklin Institute have also been received:

Recent Progress in the Science of Aeronautics. By JOSEPH S. AMES.

Spectroscopy in the Past and in the Future. By Prof. CHARLES FABRY.

Military Aircraft and their Use in Warfare. By Major-General MASON M. PATRICK.

Practical results of the Theoretical Development of Chemistry. By Prof. F. HABER.

The Steam Turbine—as a Study in Applied Physics. By Sir CHARLES A. PARSONS.

Lightning. By F. W. PEEK, Jr.

The Carbon Atom in Crystalline Structure. By Sir WM. HENRY BRAGG.

The Influence of J. Willard Gibbs on the Science of Physical Chemistry. By F. G. DONNAN.

The following Journals have been added to those now regularly received, the

list of which is given in the Report of the Council for the year ending 31st December, 1922:

Monthly Notices of the Royal Astronomical Society.
Journal of the Franklin Institute.
Scientific Papers of the Institute of Physical and Chemical Research, Tokyo.
Japanese Journal of Physics.
British Journal of Physiological Optics.

MEMBERSHIP

During the year fourteen Members, one Corporate Member, and five Student Members were elected. Twelve resignations were tendered. There is therefore an increase of eight in the membership of the Society, leaving the number of members on the 1st January, 1926 as follows:

Honorary Fellows	10
Fellows	91
Fellows non-resident in British Isles	10
Members	142
Corporate Members	9
Members non-resident in British Isles	23
Corporate Members non-resident in British Isles	2
Student Members	6
TOTAL	293

TREASURER'S REPORT FOR THE YEAR 1925

IN the Treasurer's report to the Council on the finances of the Society at the time when the reductions of the subscriptions, which have since been approved, were under consideration, it was stated that the normal expenditure of the Society was about £700 per annum, and that the subscription income from the then membership was about £680, while other sources of income might be about £250. The membership increased steadily from 1915 to 1920, but since that date it has steadily diminished. Should the membership continue to diminish, the usefulness of the Society will be greatly reduced, however satisfactory the balance sheet may appear, but if on the other hand the membership increase and approach that of 1920, the subscriptions could be safely reduced to those now in force, and still leave a safe margin between income and normal expenditure. There has already been an increase in the applications for membership since the reduced subscriptions were announced, and this must continue if the finances are to remain sound. The reserve fund now stands at just over £1000 and is sufficient to meet the extraordinary expenditure which, as stated below, will be incurred during 1926 and any deficit

likely to occur in the next year or two, but an increase in membership sufficient to balance income and normal expenditure must take place in the next few years if the Society is to continue to flourish.

The expenditure in 1925 was normal; the subscription income was low owing to more subscriptions being in arrears at the end of 1925 than at the end of 1924. The income from sales of publications was the highest yet reached, and included £12 from the Faraday Society, being the Optical Society's share of receipts from the sale of the report of the Microscope Symposium held in 1920, and £1, being a donation from the Rev. Sir J. C. W. Herschel, Bart., towards the expenses of the Herschel number of the *Transactions*.

The Council decided to subscribe £200 to the guarantee fund of the Optical Convention (1926) and to advance this sum (free of interest) to meet the preliminary expenses of the Convention until its normal income was received. This sum, less any call on the guarantee fund, will be repaid after the convention. This advance absorbed the surplus income for the year, and no further investments have been made.

The index to the first 25 Volumes of the *Transactions* is not yet complete; the cost of this together with any call on the guarantee fund of the Optical Convention will be extraordinary expenditure for 1926.

The books and manuscripts of the Society are insured against loss by fire with the Guardian Assurance Co., Ltd., for the sum of £300, but they are not included in the assets of the Society shown in the balance sheet.

E. O. Henrici,
Hon. Treasurer

16th January, 1926.

INCOME AND EXPENDITURE ACCOUNT FOR THE YEAR ENDING 31ST DECEMBER, 1925

Dr EXPENDITURE	£	s.	d.	INCOME	£	s.	d.	£	s.	d.	*Cr*
To Printing and stationery	512	13	6	By Subscriptions:							
Clerical assistance	107	10	0	Fellows, arrears	6	6	0				
Postage and receipt stamps	35	18	7½	,, for 1925	277	4	0				
Sundries	25	0	1	Members, arrears	9	9	0				
Advertising	12	0	0	,, for 1925	280	7	0				
Physical Society—share of exhibition expenses	4	5	10	Corporate members for 1925	31	10	0				
Balance—income over expenditure	201	0	7½	Students for 1925	3	2	6				
				Entrance fees	4	4	0				
					612	2	6				
				Less rebate to members of the Institute of Physics	15	0	6				
								597	2	0	
				Sales of publications				215	7	8	
				Advertisements in *Transactions*, less expenses				34	18	9	
				Interest on investments and deposit				51	0	3	
	£898	8	8					£898	8	8	

BALANCE SHEET AT 31ST DECEMBER, 1925

LIABILITIES	£	s.	d.	£	s.	d.	ASSETS	£	s.	d.
To Sundry Creditors				137	12	4	By Sundry Debtors	46	16	0
Accumulated Funds:							Optical Convention	200	0	0
Balance at 31st Dec. 1924	799	19	5½				Cash at Bank	25	14	3
Plus income over expenditure	201	0	7½				Cash in hand	8	5	0
				1001	0	1	Petty cash in hand	12	2	11
Life composition fund				52	10	0	Investments at cost*:			
Subscriptions received in advance				9	6	0	£100 Grand Trunk Western 4 % Bonds	98	10	0
							£800 War Stock 1929/47	809	0	3
	£1200	8	5					£1200	8	5

* *Mem.* Value of Investments, market price, 31st Dec. 1925 is £881. 10*s.* 0*d.*

E. O. HENRICI,
Hon. Treasurer.

We, the undersigned Auditors elected at the General Meeting held on 21st January, 1926, do hereby certify that we have examined the above accounts and do find them correct.

R. KINGSLAKE,
T. M. ROSE,
Hon. Auditors.

31*st January*, 1926.